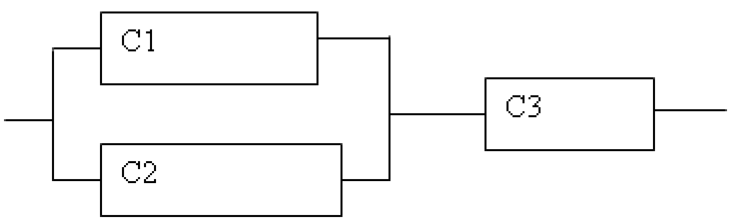
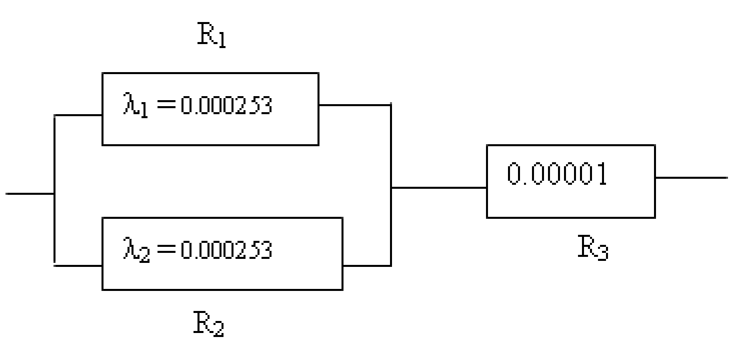
Reliability of Technical Systems

Tutorial 8 (Solution)

Q1) Below is a system with three components : C1, C2, and C3. Component C1 and C2 are independent, and identically distributed with sane CFR. The failure rate for C1 and C2 is 0.000253 (1/hour). Component C3 is also distributed with CFR and its failure rate is 0.00001 (1/hour). Estimate the overall system reliability at 1000 hours and MTTF.





For a system with CFR,

$$R(t) = \exp\left[-\int_0^t \lambda dt\right] = e^{-\lambda t}, \qquad t>0$$

$$R_1 = R_2 = e^{-0.000253\times 1000} = 0.776$$

$$R_3 = e^{-0.00001\times 1000} = 0.99$$

Tutorial 8 (Solution)

$Rs = [\ 1- (1-R_1)(1-R_2)]\ R_3$

$= [\ 1- (1-0.776)^2].99$

Rs = 0.940

$$R_s = (2e^{-\lambda 1 * t} - 2e^{-2\lambda 1 * t})e^{-\lambda 3 * t}$$

MTTF= $\int_0^\infty R_s(t)dt$

$$= \frac{2}{\lambda 1 + \lambda 3} - \frac{1}{2\lambda 1 + \lambda 3}$$

= 5666 hours

Tutorial 8 (Solution)

Q2) Consider a pile foundation, in which pile groups are used to support the individual column footings. Each of the pile group is designed to support a load of 200 tons. Under normal condition, this is quite safe. However, on rare occasions the load may reach as high as 300 tons. The engineer wishes to know the probability that a pile group can carry this extreme load of up to 300 tons.

Based on previous experiences with similar pile foundations, the engineer estimated a probability of 0.70 that any pile group can support a 300-ton load. And, among those that have capacity less than 300 tons, 50% failed at loads less than 280 tons.

To improve the estimated probability, the engineer orders one pile group to be proof-loaded to 280 tons.

If the pile group survives the specific proof load, what is the probability that the pile group can support a load of 300 tons ?

Let A: event that the capacity of pile group >= 300 tons
T: event of a successful proof load

$$P(\bar{T}|\bar{A}) = 0.5$$

P(A) = 0.70
P(T|A) = 1

$$P(A|T) = \frac{P(T|A)P(A)}{P(T|A)P(A) + P(T|\bar{A})P(\bar{A})} = \frac{(1.00)(0.70)}{(1.00)(0.70)+(0.5)(0.3)}$$

= 0.824