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# Reliability of Technical Systems

# Tutorial 7 (Solution)

Assume a 3oo4 (3 out of 4) system with identical components.

Q1) Calculate the failure likelihood QS of the system, while the failure likelihood of the components is given qi = 0.01, i = 1, 2, 3, 4.

This is a question regarding failure probability without considering the DF.
3oo4 means we need at least 3 non-failure components in order to make the system function.

$$p^4 + 4p^3q + 6q^2p^2 + 4pq^3 + q^4 = 1$$

$$Q_S = 6q^2p^2 + 4pq^3 + q^4 = 6q^2(1-q)^2 + 4(1-q)q^3 + q^4 = 3q^4 - 8q^3 + 6q^2$$

If all failures are independent of each other, arises with $q_i = 0.01$:

$$Q_S = 5.92 \cdot 10^{-4}$$

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Assume a 3oo4 (3 out of 4) system with identical components.

Q2) Determine system failure likelihood . Please take into account dependent failures with the help of the β-factor-model (β = 10%). The observed failures of the components lead to the failure likelihood qj = 0.01, j = 1, 2, 3, 4.

This is a question regarding failure probability considering the DF.

For 'm-out-of-n-system' it is generally: $Q_t = Q_1 + Q_n$

Definition of the $\beta$ – factor:

$$\beta = \frac{Number\ of\ DF}{Number\ of\ all\ failures} \qquad \beta = \frac{Q_n}{Q_1 + Q_n} = \frac{Q_n}{Q_t}$$

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Finally,

$$Q_k = \begin{cases} (1-\beta)\cdot Q_t & k=1 \\ 0 & m>k>1 \\ \beta\cdot Q_t & k=n \end{cases}$$

$$Q_{S;DF} = (3Q_1^4 - 8Q_1^3 + 6Q_1^2) + Q_4 \approx 6Q_1^2 + Q_4$$

K=1

K=4

$$Q_k = \begin{cases} (1-\beta)Q_t, & \text{if } k=1 \\ \beta Q_t, & \text{if } k=n=4 \end{cases}$$

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$$Q_{S;DF} \approx 6Q_1^2 + Q_4 = 6(1-\beta)^2 Q_t^2 + \beta Q_t$$

$$Where\ Q_t = 0.01\ and\ \beta = 0.1$$

(Qt is the total failure likelihood of one component )

Therefore,

$$Q_{S;DF} \approx 6(1-0.1)^2(0.01)^2 + 0.01 \times 0.1 \quad =$$

$$4.86 \times 10^{-4} + 10^{-3} \approx 1.483 \times 10^{-3}$$