Reliability of Technical Systems

Tutorial 6 (Solution)

Q1) A driver is traveling alone with a SUV in the desert. What is the probability that the vehicle stops with a flat tire, if only one spare tire is carried by this driver? In this question, you can assume that no extra spare tires are available. Here are the data you could use for solving this question.

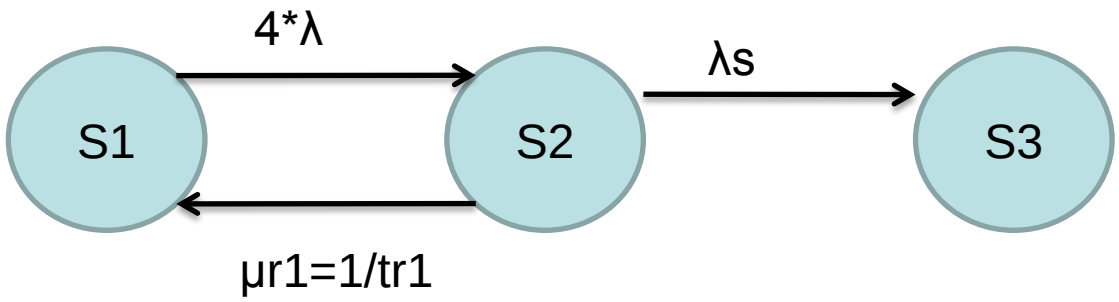
Failure rate of one tire : $\lambda = 10^{-4}$ [1/hour]
Failure rate of the spare tire: $\lambda S = 10^{-3}$ [1/hour]
Repair duration (time to install the spare tire): $tr1 = 2$ hours

* *Hint: You may assume the descriptions of these questions can be regarded as steady-state behaviors*.

Tutorial 6

Solution:

First we need to define different states :



State 1 (S1) : Four tires are in good condition

State 2 (S2): One tire becomes flat (Three tires are in good condition)

State 3 (S3) : spare tire become flat (Three tires are still in good condition)

Calculation of the Steady-State State Probabilities

In the state model with constant transition rates (homogeous Markovian model) the state probabilities p_i approach their final values P_i.

These values express the steady-state behaviour:
For all states the sum of "inputs" is equal to the sum of "outputs".

$$\forall\ j \in \{1, \ldots, m\}: \quad \sum_{i=1}^{m} P_i \cdot \alpha_{i,j} = \sum_{k=1}^{m} P_j \cdot \alpha_{j,k}$$

This formula expresses a **homogenous linear system of equalities** to be solved under the condition

$$\sum_{i=1}^{m} P_i = 1$$

(1^{st} Method to calculate the steady-state state probabilities).

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$$-P1 \times (4\lambda) + P2 \times (\mu r1) = 0$$
$$-P2 \times (\lambda s) - P2 \times (\mu r1) + P1 \times (4\lambda) = 0$$
$$P1 + P2 + P3 = 1$$

P1=0
P2=0
P3=1

What these numbers mean ???

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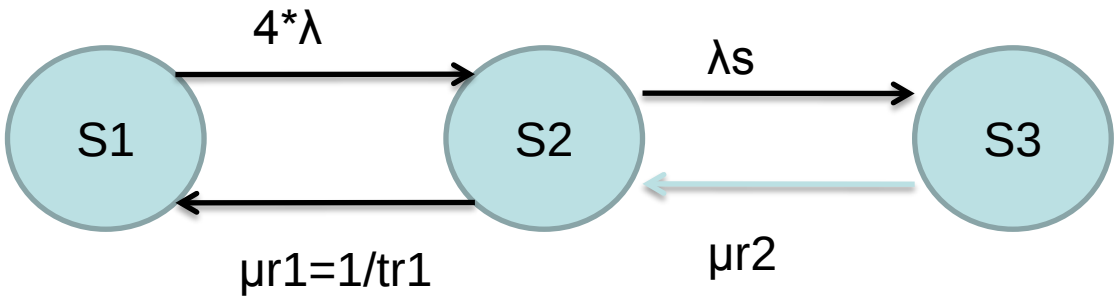
Q2) It takes about a week for the driver to go to shop and bring new spare tire (you may assume repair rate is about $6 \cdot 10^{-3}$ (1/hour)). In this case, what is the probability that the vehicle stops with a flat tire, assuming other parameters remain the same as the Q1.

Failure rate of one tire : $\lambda = 10^{-4}$ [1/hour]
Failure rate of the spare tire: $\lambda S = 10^{-3}$ [1/hour]
Repair duration (time to install the spare tire): $tr1$ = 2 hours
Repair rate of spare tire: $\mu r2 = 6 \cdot 10^{-3}$ [1/hour)]

Tutorial 6



State 1 (S1) : Four tires are in good condition

State 2 (S2): One tire becomes flat (Three tires are in good condition)

State 3 (S3) : Spare tire become flat (Three tires are still in good condition)

Tutorial 6

$$-P1 \times (4\lambda) + P2 \times (\mu r1) = 0$$
$$-P3 \times (\mu r2) + P2 \times (\lambda s) = 0$$
$$P1 + P2 + P3 = 1$$

P1=0.99907
P2=7.9925E-4
P3=1.3321E-4