

# Reliability of Technical Systems



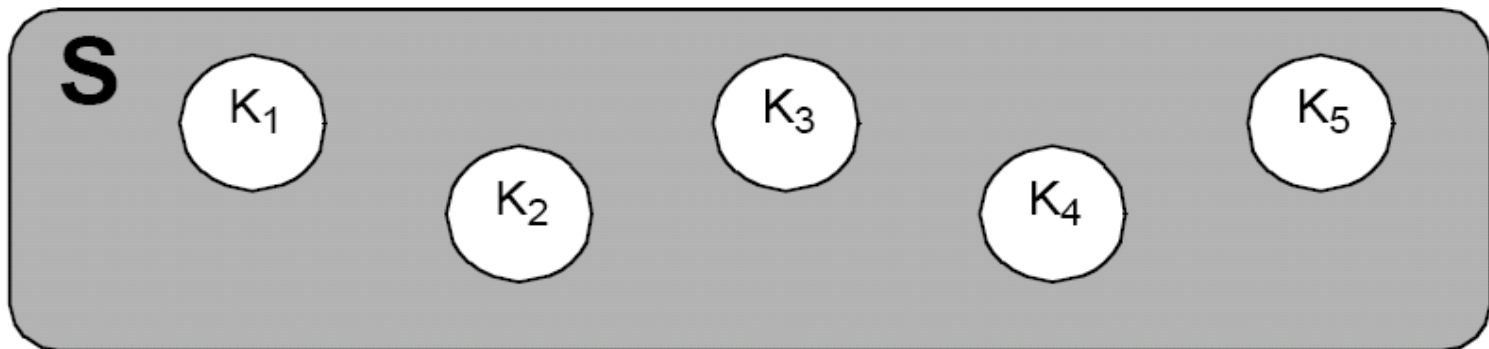
# Main Topics

1. Introduction, Key Terms, Framing the Problem
2. Reliability Parameters: Failure Rate, Failure Probability, etc.
3. Some Important Reliability Distributions
4. → **Component Reliability**
5. Software Reliability
6. Fault Tolerance
7. System Reliability I
8. Dependent Failures
9. System Reliability II
10. Static and Dynamic Redundancy
11. Advanced Methods for System Modelling and Simulation I
12. Advanced Methods for System Modelling and Simulation II
13. Human Reliability

# Component Reliability and System Reliability

system  $S = \{K_1, \dots, K_n\}$ ,

number of components:  $n$



Conclusions if the system structure is known:

**Reliability of  $K_1, \dots, K_n \Leftrightarrow$  Reliability of  $S$**  (both directions)

Assumption:

Failures of the components are **stochastically independent**.

## Relationship Between $z(t)$ and $R(t)$

By integration of  $z(t) = \frac{f_L(t)}{R(t)} = -\frac{dR(t)}{dt} \cdot \frac{1}{R(t)}$  we obtain a direct relationship between the failure rate and the reliability:

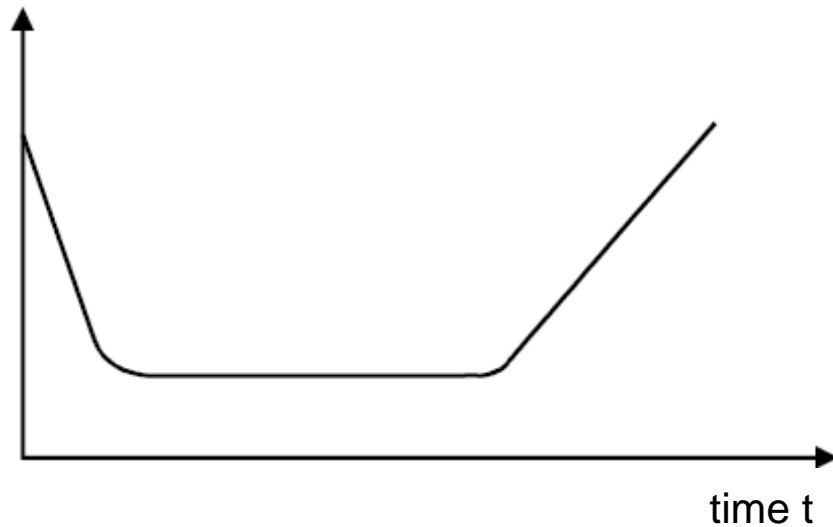
$$\int_0^t z(x) dx = -\ln(R(t)) + C$$

The integration constant is  $C = 0$  because of  $R(0) = 1$ . By exchanging the two sides of this equation and an exponentiation we obtain

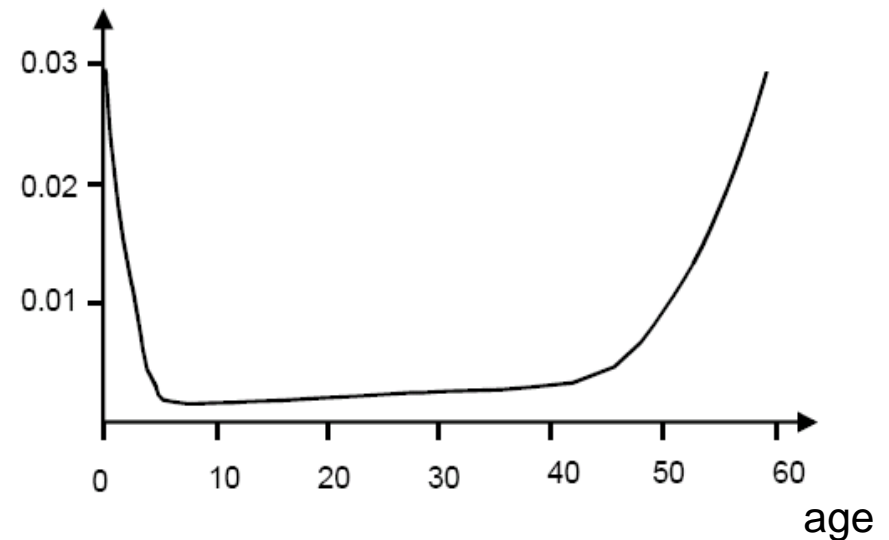
$$R(t) = e^{-\int_0^t z(x) dx} = \frac{1}{e^{\int_0^t z(x) dx}} \quad \text{if } z(t) = \lambda \Rightarrow R(t) = \frac{1}{e^{\lambda \cdot t}}$$

## Comparison Between Failure and Mortality Rate

failure rate  $z(t)$



mortality rate

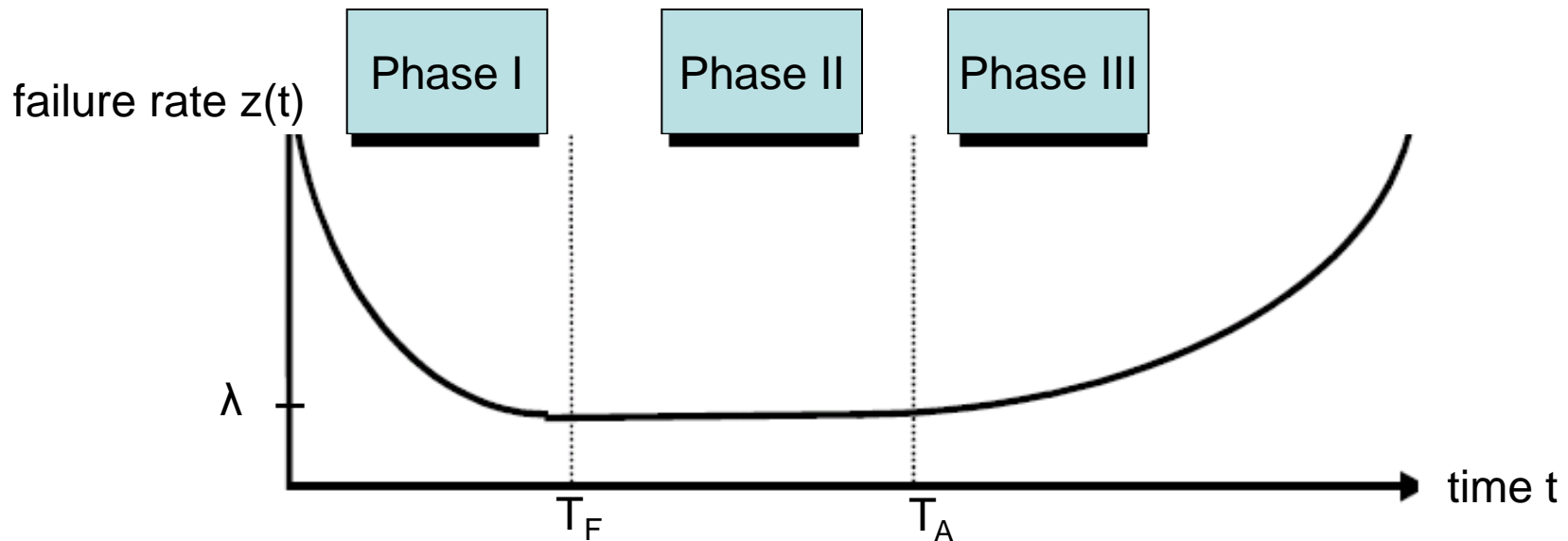


**Failure rate:** Probability per  $dt$  that a unit that survived until time  $t$  will fail in the next time interval.

**Mortality rate:** Probability at a given age to die within the next year.

## Bathtub Curve (I)

The bathtub curve describes the live time of a component.



Phase	Failure Rate	Meaning
1	decreasing	Phase of <i>early failures</i> (“optimising phase”)
2	constant	Phase of <i>random failures</i> (no systematic failures)
3	increasing	Phase of <i>late failures</i> (e.g. wear-out, aging, etc.)

## Failure (Fault):

Wrong or "missing" function of a component.

### Failure causes:

- Design failure
- Manufacture failure
- Operation failures
  - ✓ Failures due to disturbances
  - ✓ Random physical failures
  - ✓ Handling failures
  - ✓ Maintenance failures
  - ✓ Wearing failures

During the whole time random faults occur (with failure rate  $\lambda$ ) because of disturbance from the environment or random physical processes within the component.

In addition, early faults contribute to an increased failure rate for a period of some weeks. Early faults can be considered as the "children's diseases" of a hardware component. Their origin lies in undetected manufacturing weaknesses, which will cause a failure after a period of operation under stress.

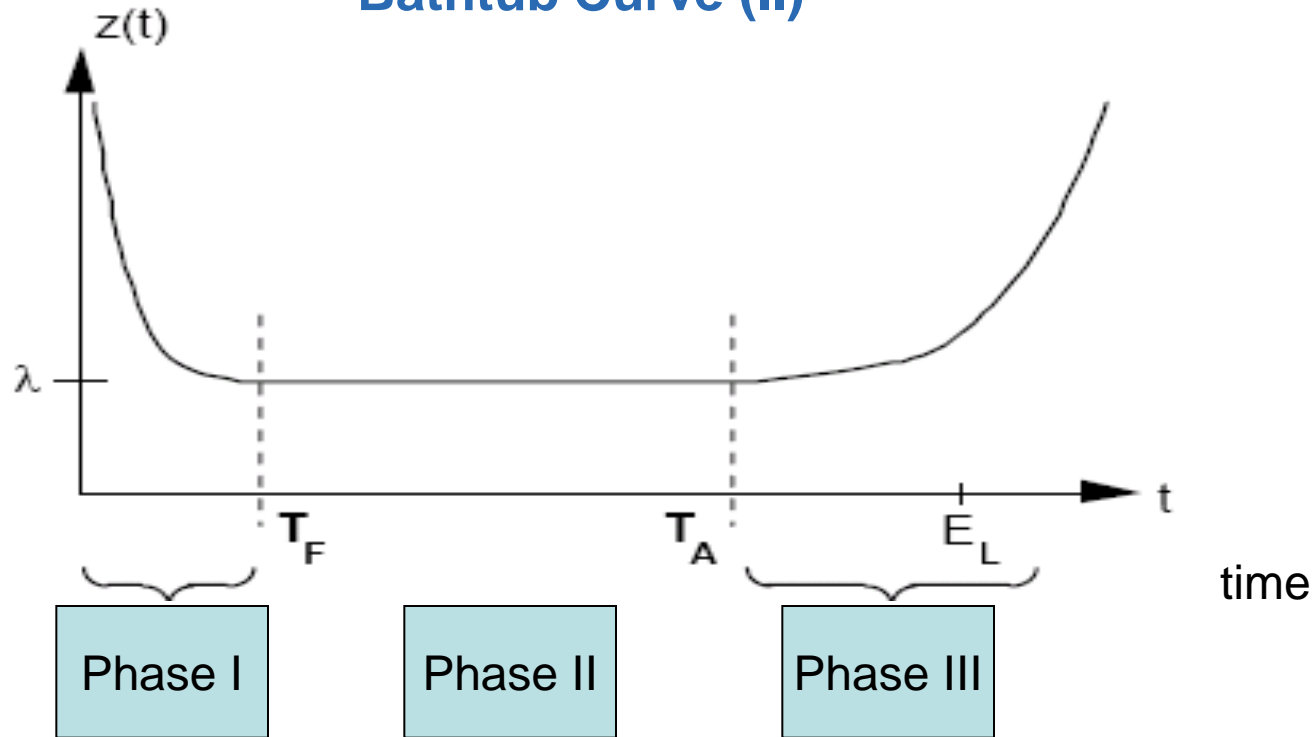
After the "retiring age" is reached (typically before the mean time to failure), wear-out faults occur in addition to the random faults. In the case of semiconductor components the main cause for aging lies in the temperature changes due to the switching on and off of the supply voltage. While early faults and random faults can be neglected sometimes, wear-out faults have a dominant influence on the failure rate.

Early faults can be modeled by a Weibull distribution, wear-out faults by a normal distribution (or a logarithmic normal distribution), and the phase in between where only random faults occur can be modeled by an exponential distribution.



failure rate

## Bathtub Curve (II)



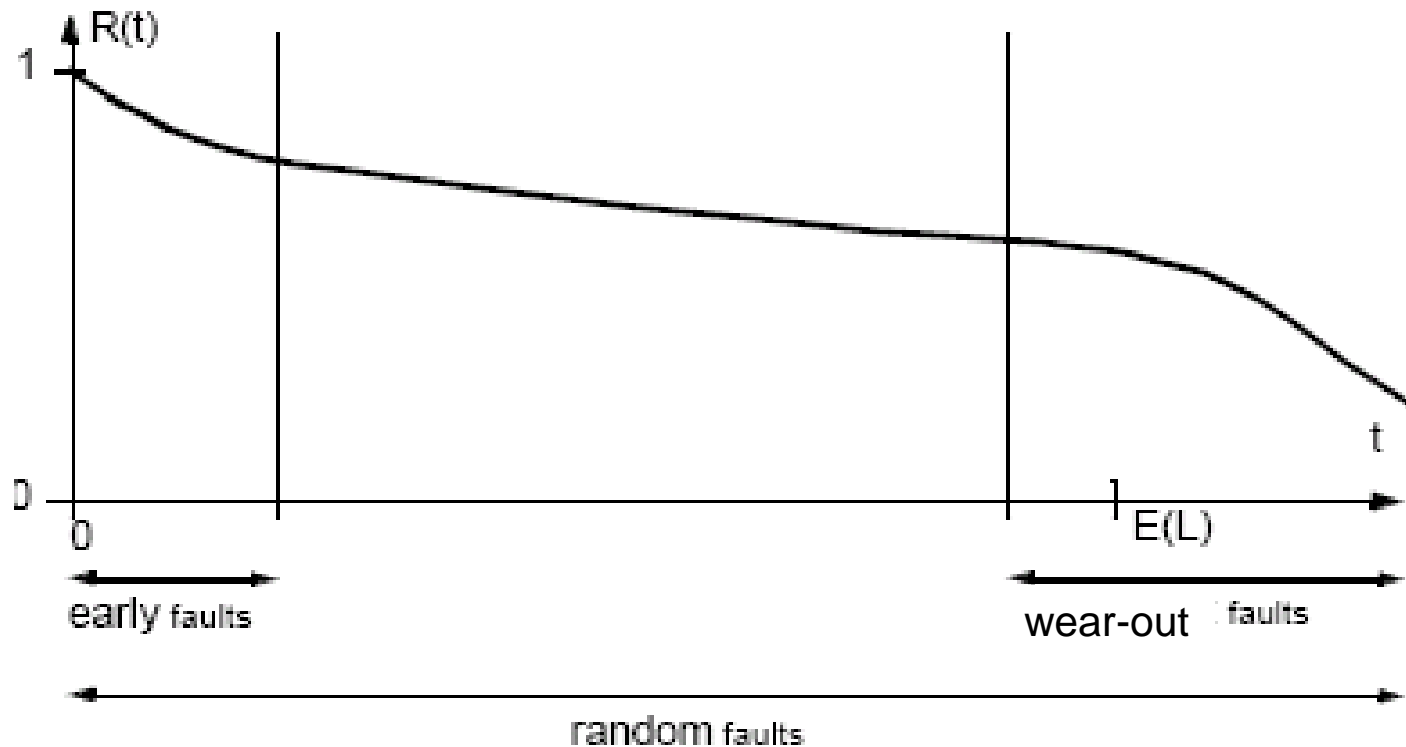
$[0, T_F]$ :  $R(t) = e^{-\frac{1}{\alpha} t^\beta}$  Weibull distributed with  $\beta < 1$ ,  $T_F =$  some weeks.

At time  $T_F$ :  $z(t) = \frac{\beta}{\alpha} \cdot t^{\beta-1} = \lambda$

$(T_F, T_A)$ : Exponential distributed with  $\beta = 1$ ,  $z(t) = \lambda$  (constant).

$[T_A, \infty)$ : Normal distributed (WD with  $\beta \approx 3,44$ ; or Log Normal),  $T_A =$  years.

## Bathtub Curve and Reliability



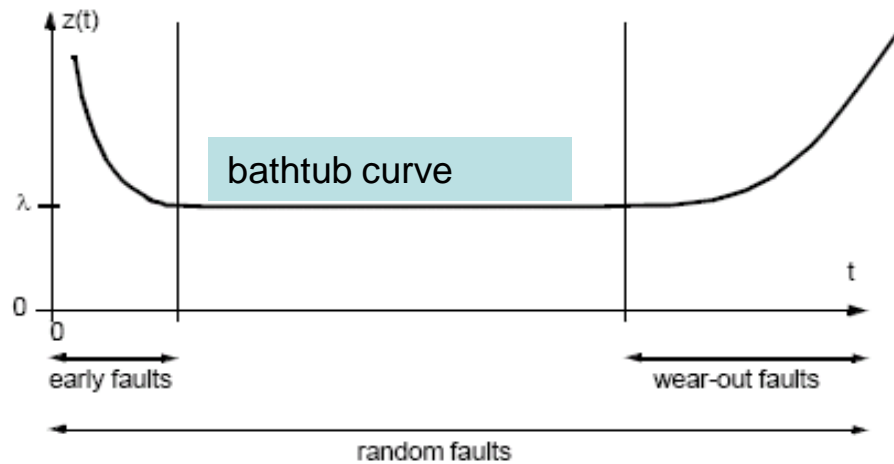
The three time ranges with different behaviour of the failure rate have an influence on the reliability (probability to survive) as follows:

- In the beginning where both early and random faults are active the reliability is somewhat more reduced than in the next phase.
- In the (relatively long) medium phase the reliability is only influenced by the exponential distribution of the random faults.
- In the late phase the reliability decreases rapidly down to zero due to wear-out.

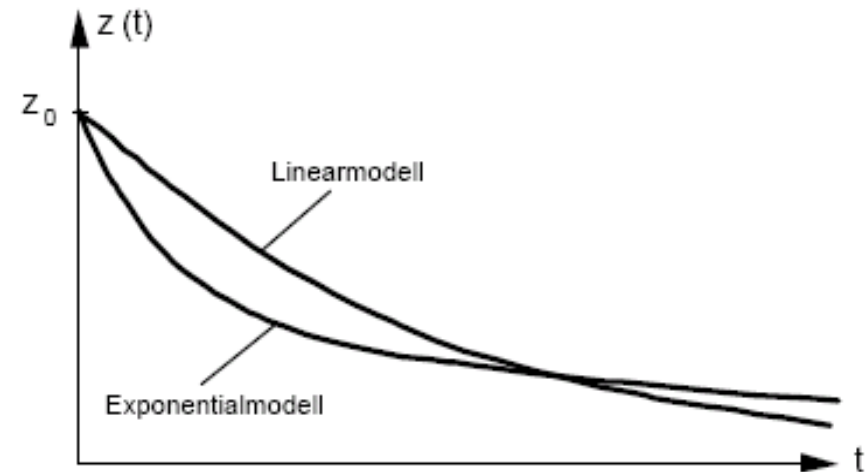
For some type of components the late phase typically begins after many decades, when the system is out of use anyway. Therefore the exact modelling of this phase is not of interest for such components.

If the early faults do not have a strong influence (as is the case for many hardware components), then the modeling by a pure exponential distribution is justified.

## Failure Rates of Various Unit Types



Hardware / Mechanic Components



Software

### Comments:

The hypothesis of a constant failure rate has to be verified for each case by statistical method. It holds true if, and only if, the probability model of an exponential distribution is correct.

Non constant failure rates implicate another probability model, e.g. the Weibull distribution.

## How Phase I and Phase III can be reduced?

### ➤ Phase I: **Pre-aging („Burn-in“)**

**pre-aging:** Components are operated in a test environment under realistic or even an increased stress (**burn-in**) before they are really used.

### ➤ Phase III: **Preventive Replacement**

**preventive replacement:** Components are replaced before they reach the late phase – even if they are still faultless.

Preventive replacement enables periodical maintenance, where typically multiple "old" components are replaced at a time. This procedure can cause less cost than replacement on failure occurrence.

## Pre-aging

Examples of pre-aging:

- pre-aging of electric bulbs for traffic lights.
- burn-in of semiconductor components by operation under an increased temperature.

Preventive replacement is mainly applied to components where the wear-out failure rate strongly increases after some time (typically for mechanical components).

The prediction or assessment of reliability is actually an evaluation of unreliability, the rate at which failures occur. The nature and underlying cause of failures must be identified and corrected to improve reliability. Reliability data consist of reports of failures and reports of duration of successful operation of the monitored equipment/system.

Reliability data is used for three main purposes:

- (1) To verify that the equipment is meeting its reliability requirements
- (2) To discover deficiencies in the equipment to provide the basis for corrective action
- (3) To establish failure histories for comparison and for use in prediction

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## Reliability Data

Reliability or failure data can be obtained from a number of sources.

The most useful are:

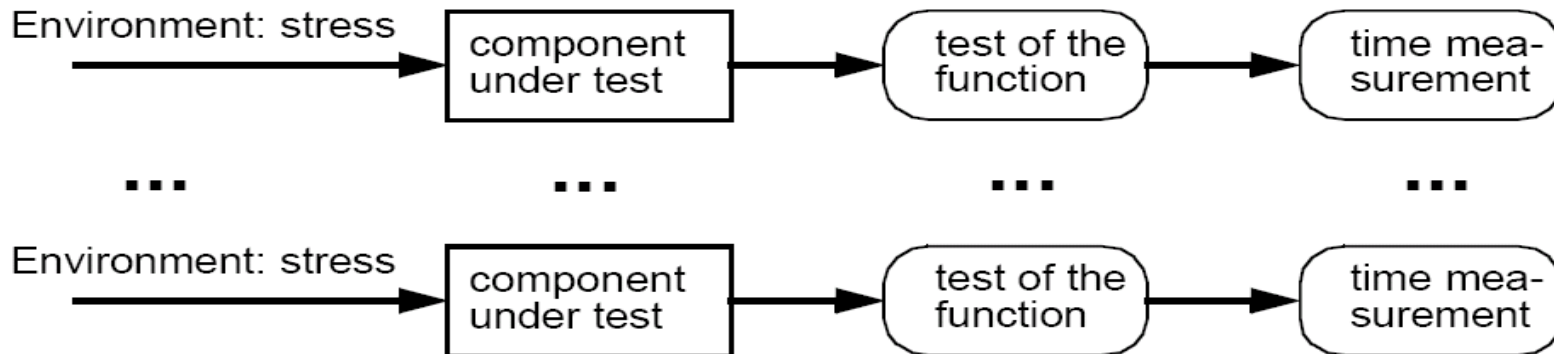
- Field data
- Reliability databases (OREDA, GIDEP, etc)
- Reliability test (experiment) data

The other sources are in most cases incomplete.



## Reliability Experiment

Experiment for the determination of reliability measures:



- Realistic stress of several components
- Measurement of the time to failure

For statistical reasons a sufficient number of components is needed for the reliability experiment.

The stress from the environment should be similar to the stress expected during future operation with respect to voltage, temperature, input signals, etc..

The test states when the corresponding component fails (failure of its function).

## Determination of the failure rate

If in the reliability experiment the test of  $n$  components over a period  $t$  leads to  $k$  failures

then the failure rate can be estimated by  $\lambda = \frac{k}{n \cdot t}$ .

Under the assumption of a **constant failure rate** it can be concluded that the time to failure is **exponentially distributed**.

This enables the quantification of all other measures by the formulae

The conclusion from the reliability experiment is relatively simple, as can be seen from the evaluation formula.

With the known statistical methods also the confidence intervals can be determined.

## Practical Problems of the Reliability Experiment

- What is a **realistic stress** ?
- Does a realistic stress of **software** exist by some input data ?
- Can we choose an increased stress for a **worst case estimation** ?
- **Which function** should be required from the components under test ?
- Do the **tests** cover all malfunctions ?
- What is the cost of this component **destroying** experiment ?
- How can the **number of components** under test be minimized ?
- **Duration** of the reliability experiment ?
- Are there techniques for a **speed-up** of the experiment ?

## Malfunctions of an unit (failure modes)

<b>Functions</b>	<b>Types of failure</b>
Closing	Fails open Only partly closed
Opening	Fails closed Only partly opened
Remain closed	Opens completely Partly opens
Remain opened	Closes completely Partly closes

## Techniques to speed up the experiment

- Sequential test
- Accelerated test
- Extrapolation

## Sequential Test (I)

If the actual failure rate  $\lambda$  does not exceed the limit  $\lambda_1$  ( $\lambda < \lambda_1$ ) with high probability  $w_1$ , the components are to be accepted.

On the other hand, if  $\lambda$  does not under-run the limit  $\lambda_2$  ( $\lambda > \lambda_2$ ) with high probability  $w_2$ , the components are to be rejected.

Given:  $\lambda_1$ , sample size  $n$ , acceptance threshold  $k$ , time of experiment  $t$ .

Let  $X$  be the number of failures (Poisson distributed) within the time interval  $[0, t]$ .

$$p(X \leq k) = \sum_{i=0}^k \frac{(n \cdot \lambda_1 \cdot t)^i}{i!} e^{-n \cdot \lambda_1 \cdot t}$$

## Sequential Test (II)

For given probabilities  $\alpha$  and  $\beta$  is true:

	<u>For <math>\lambda = \lambda_1</math></u>	<u>For <math>\lambda = \lambda_2</math></u>
Probability of acceptance	$1 - \alpha$	$\beta$
Probability of rejection	$\alpha$	$1 - \beta$

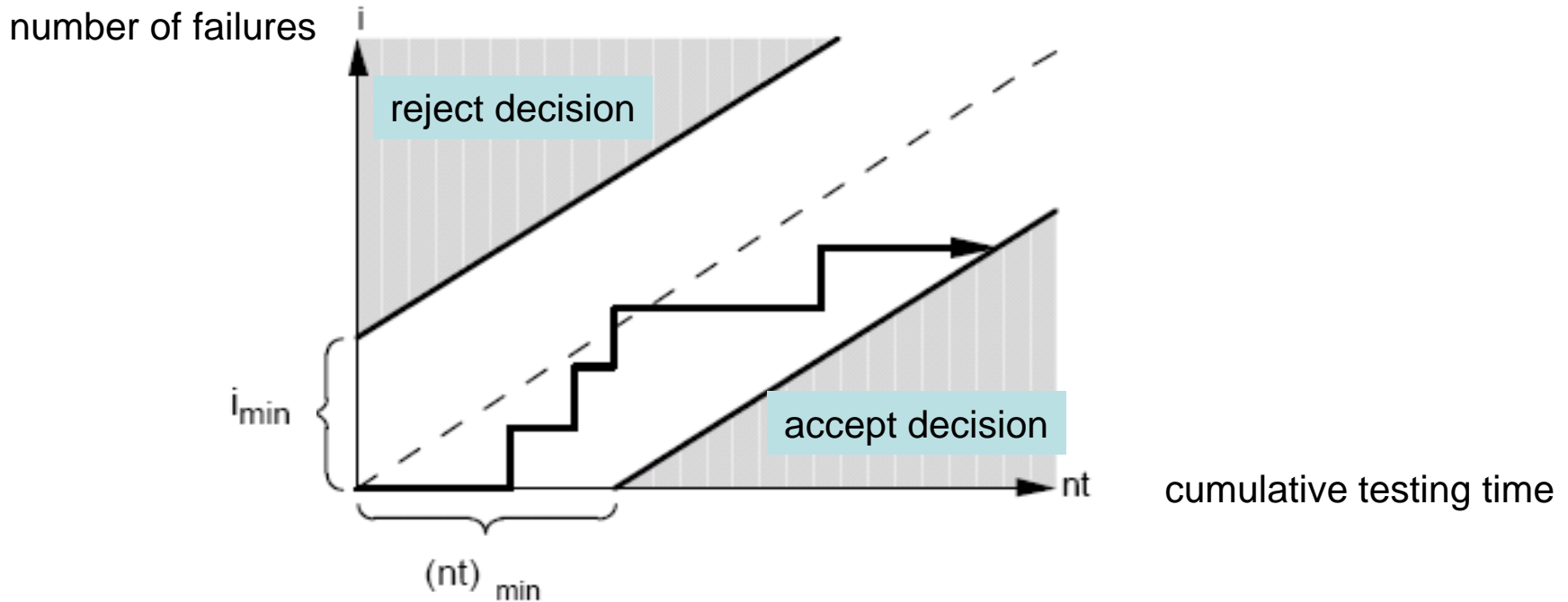
Accept decision if:

$$i \leq \frac{\ln \frac{\beta}{1 - \alpha}}{\ln \frac{\lambda_2}{\lambda_1}} + \frac{\lambda_2 - \lambda_1}{\ln \frac{\lambda_2}{\lambda_1}} \cdot n \cdot t$$

Reject decision if:

$$i \geq \frac{\ln \frac{1 - \beta}{\alpha}}{\ln \frac{\lambda_2}{\lambda_1}} + \frac{\lambda_2 - \lambda_1}{\ln \frac{\lambda_2}{\lambda_1}} \cdot n \cdot t$$

## Sequential Test (III): Illustration



The minimum testing time  $(nt)_{\min}$  and the minimum number of failure  $i_{\min}$  amount to:

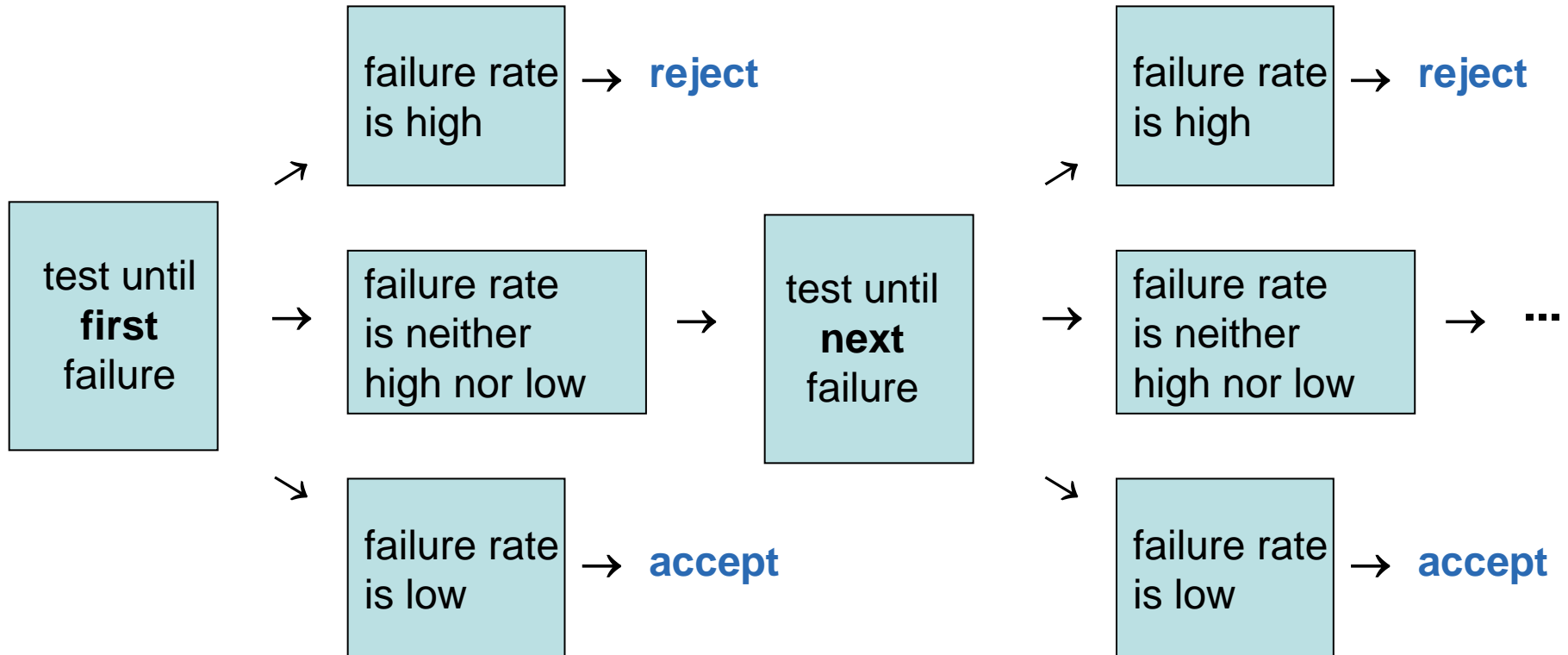
$$(n \cdot t)_{\min} = \frac{\ln \frac{\beta}{1 - \alpha}}{\lambda_1 - \lambda_2}$$

$$i_{\min} = \frac{\ln \frac{1 - \beta}{\alpha}}{\ln \frac{\lambda_2}{\lambda_1}}$$

where  $\alpha$  and  $\beta$  are probabilities



## Sequential Test (IV): Algorithm



## Extrapolation (I)

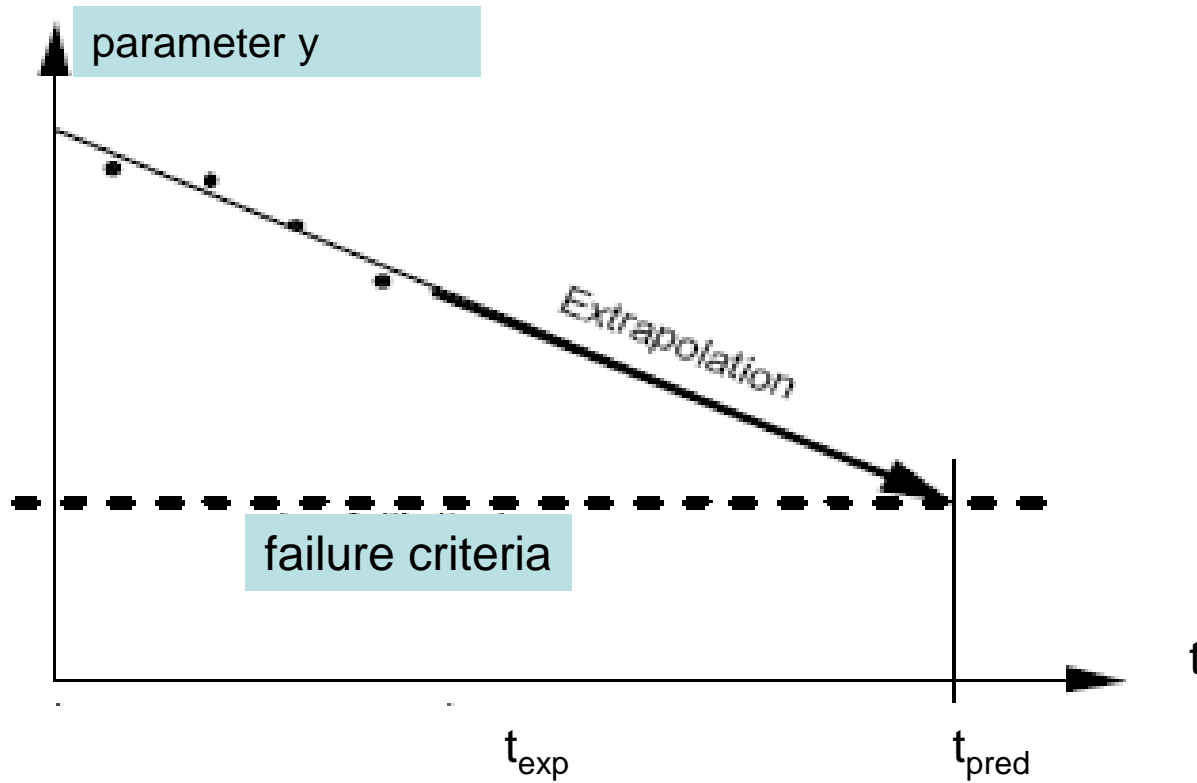
Failure prediction by extrapolation:

- ✓ Shorter testing time
- ✓ Test is non-destructive

Conditions:

- ✓ **Just drift failures**
- ✓ **Failure criteria are known**

## Extrapolation (II): Illustration



$t_{exp}$  testing time

$t_{pred}$  predicted life time (forecast through extrapolation)

## Accelerated Test (I)

The Arrhenius acceleration model is widely used to predict life as a function of temperature. It applies specifically to those failure mechanisms that are temperature related and which are within the range of validity for the model.

It states that : 
$$\text{Life} = A(e)^{\frac{E}{kT}} \quad (8.43)$$

where:

- Life = a measure of life e.g., median life of a population of parts
- A = a constant determined by experiment for the parts involved
- e = the base of the natural logarithms
- E = activation energy (electron volts - a measure of energy) this is a unique value for each failure mechanism (Examples of the activation energies for some silicon semiconductor failure mechanisms are shown in Table 8.7-1.)
- k = Boltzman's constant =  $8.62 \times 10^{-5}$  eV/K
- T = Temperature (Degrees Kelvin)

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## Accelerated Test (II): Algorithm

Acceleration factor  $F_{1,2} = \frac{L_1}{L_2} = e^{\frac{E}{K}(\frac{1}{T_1} - \frac{1}{T_2})}$

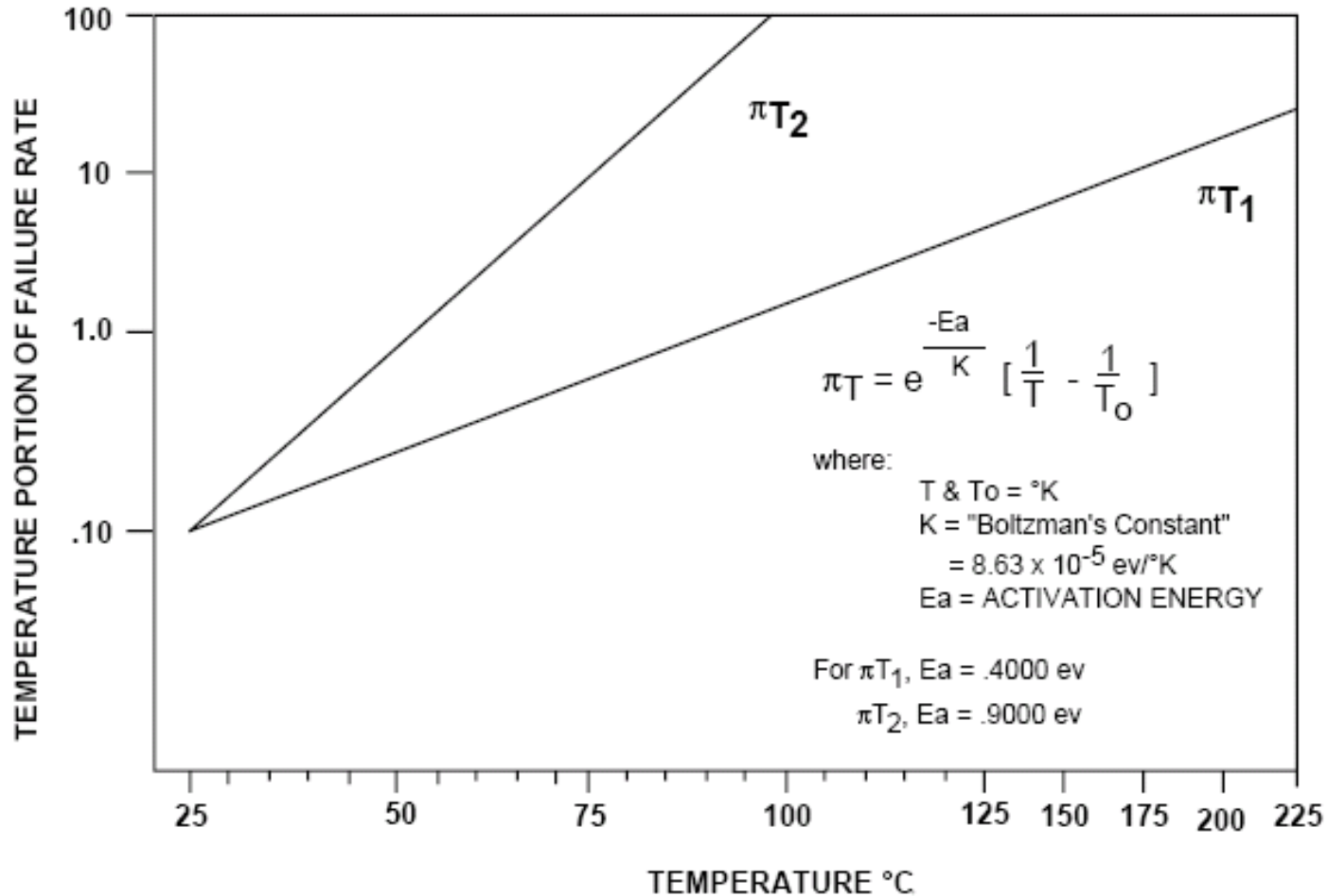
Measure mean life time  $L_2$  of component by  $T_2 > T_1$ , where  $T_1$  is a low temperature.

If the quotient  $E/K$  is unknown, repeat the test by  $T_3 > T_1$  and find it from

$$\frac{L_2}{L_3} = e^{\frac{E}{K}(\frac{1}{T_2} - \frac{1}{T_3})}.$$

Then calculate  $L_1 = L_2 \cdot e^{\frac{E}{K}(\frac{1}{T_1} - \frac{1}{T_2})}.$

## Accelerated Test (III): Illustration



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