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Reliability of Technical Systems

Main Topics

- Introduction, Key terms, framing the problem
- Reliability parameters: Failure Rate, Failure Probability, Availability, etc.

→ **Some important reliability distributions**

- Component reliability
- Software reliability
- Fault Tolerance
- System reliability: Structure and State Modelling
- Dependent failure
- Human reliability
- Static and dynamic redundancy
- Advanced methods for systems modeling and simulation

Most relevant distributions in Reliability Theorie:

- Continuous ones: **Exponetial, Weibull, Normal, Log Normal, Uniform (Rectangular)**.

- Discrete ones: **Poisson, Binomial (Bernoulli).**

Number	Name	Distribution
1		Binomial
2		Exponential
3		Log Normal
4		Normal
5		Poisson
6		Uniform
7		Weibull

Exponential distribution

The exponential distribution is the only continuous memoryless random distribution.

The exponential distribution is the most common and simplest distribution function to model the reliability of components.

The failure rate and the time to failure are reciprocal.

Application: To estimate the reliability of componets/systems with constant failure rate.

Exponentially Distributed Time to Failure

If the time to failure is exponential distributed with parameter λ, we obtain:

Density: $f_L(t) = \lambda \cdot e^{-(\lambda \cdot t)}$

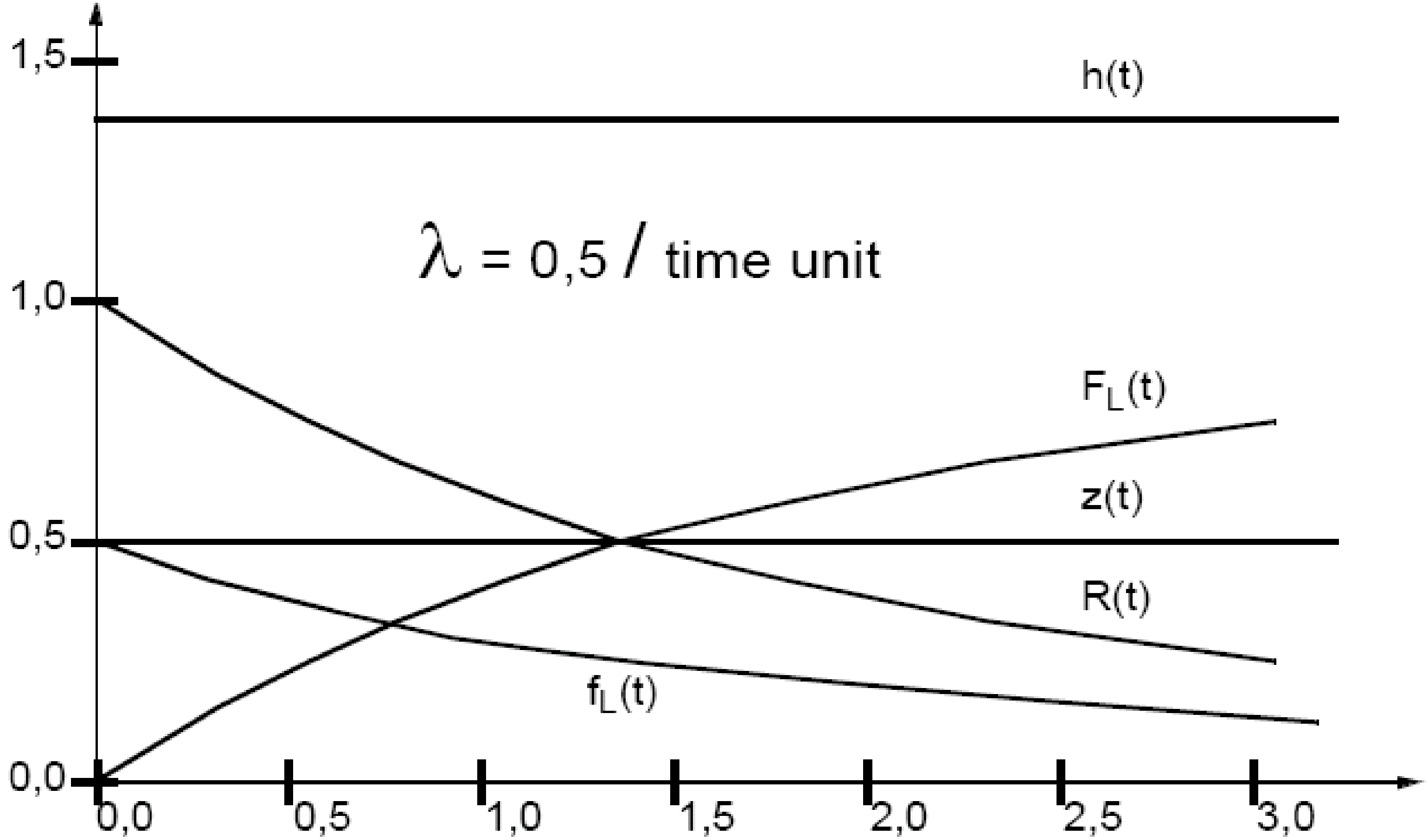
Failure prob.: $F(t) = \int_0^t (\lambda \cdot e^{-(\lambda \cdot x)}) dx = -(e^{-(\lambda \cdot x)})\Big|_0^t = 1 - e^{-(\lambda \cdot t)}$

Reliability: $R(t) = e^{-(\lambda \cdot t)}$

Mean time to f.: $E(L) = \int_0^\infty (e^{-(\lambda \cdot t)}) dx = -\left(\frac{1}{\lambda} \cdot e^{-(\lambda \cdot t)}\right)\Big|_0^\infty = \frac{1}{\lambda}$

Failure rate: $z(t) = \frac{f_L(t)}{R(t)} = \lambda$ **constant**

Illustration of the Exponentially Distributed Time to Failure



Stationary availability

Only for repairable systems the stationary availability (steady-state availability)

V is defined by

$$V = \frac{E_L}{E_L + E_R} = \frac{MTTF}{MTTF + MTTR}$$

The availability V denotes the probability, that a system is properly functioning at any point in time. The general (time dependent) availability V(t) can only be calculated after the introduction of a state model.

Mean time to failure (MTTF)	E_L or E (L)
Mean time to repair (MTTR)	E_R or E (B)

Assumption: Exponential distribution

$E_L = 1/\lambda$

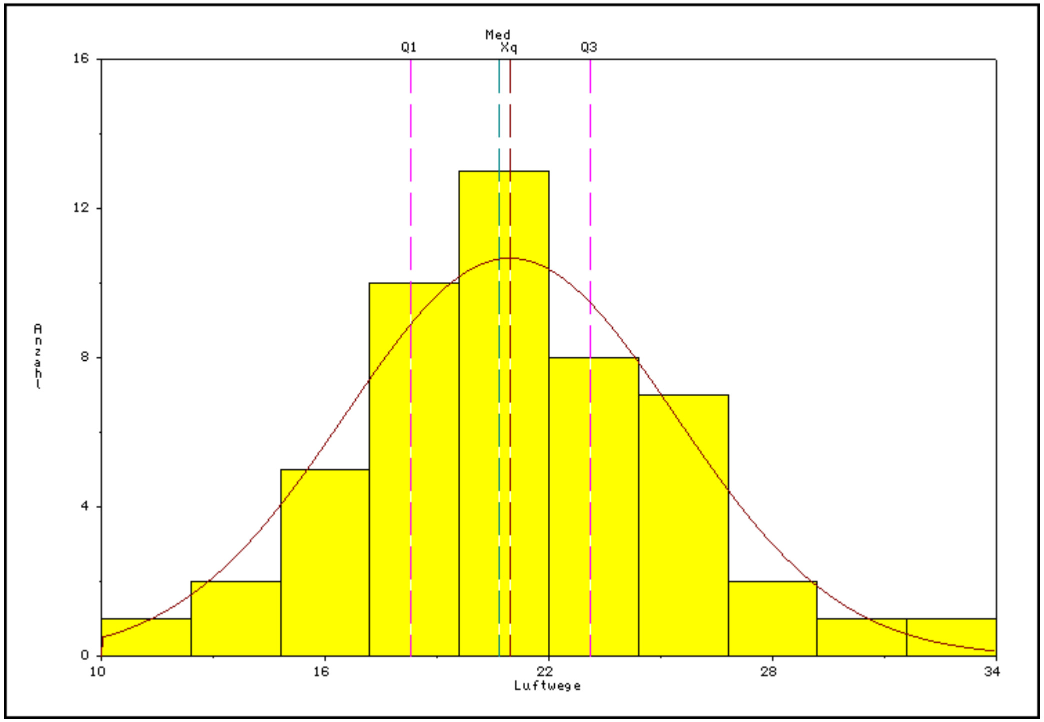
$E_R = 1/\mu$

$$V = \frac{1/\lambda}{1/\lambda + 1/\mu} = \frac{1/\lambda}{\mu + \lambda \big/ \lambda \cdot \mu} = \frac{\mu}{\lambda + \mu}$$

Where λ - failure rate,
μ – repaire rate

Statistical data and (statistical) distribution function

A histogram is the graphical version of a table that shows what proportion of cases (relative occurrence) fall into each of specified intervals. A generalization of the histogram construct a very smooth probability density function from the supplied data.



Questions the Histogram Answers

- What is the most common value?
- What distribution (center, variation and shape) does the data have?

To check with Kolmogorov`s or χ^2 criteria.

Application examples of distributions

Distribution	Field of Application
Exponential	for constant failure rate
Weibull	$\beta > 1$ for monotone increasing rates $\beta < 1$ for monotone decreasing rates
Normal	In case of statistically independent random variables
Logarithmic Normal	for time to repair
Uniform	for wear-out failures
Binomial	for n-out-of-m systems
Poisson	for statistical reliability experiment

Exercise example:

For a larger number of simultaneously implemented identical devises it was discovered after one year that 5% of the devices failed, and were not repaired.

1. What is the mean lifetime E of the devices, if an exponential distribution with parameter λ is assumed?

2. What is the mean lifetime E of the devices, if an uniform distribution with the parameters a and b is assumed and a = 0?

What assumption leads to more optimistic reliability forecast?

Solution:

1. Exponential distrbution

$$R(t) = e^{-\lambda \cdot t}$$

t=1 year, F(t)= 0.05 (5%), than R(t)=0.95

$$R(t=1) = e^{-\lambda \cdot 1 year} = 0.95, \quad \lambda = \frac{-\ln 0.95}{1 year} = 5.13 \cdot 10^{-2} / year$$

Mean lifetime $E_{exp}=1/\lambda=$19,5 years

Solution:

2. Uniform distrbution

$$R(t) = \frac{b-t}{b-a}$$

t=1 year, F(t)= 0.05 (5%), than R(t)=0.95

$$R(t) = \frac{b-t}{b-a} = \frac{b-1}{b-0} = 0.95, \quad b-1\,year = 0.95b \Rightarrow b = 20\,years$$

Mean lifetime E_{uni}=(a+b)/2=10 years

$$E_{\exp} \neq E_{uni}$$

Interaction of different distributed failure effects (I)

Assumptions:

- Failures are independent,
- Failure effects interact one after another.

In general: $$R(t) = \prod_{i=1}^{N} R_i(t)$$

By 2 failure effects: $R(t)= R_1(t)*R_2(t)$, $F(t)=1- R1(t)*R2(t)$,

$$f(t) = -\frac{d(R_1(t)\cdot R_2(t))}{dt} = -\dot{R}_1(t)\cdot R_2(t) - R_1(t)\cdot \dot{R}_2(t), \qquad E = \int_0^\infty R_1(t)\cdot R_2(t)\,dt,$$

$$z(t) = \frac{-\dot{R}_1(t)\cdot R_2(t) - R_1(t)\cdot \dot{R}_2(t)}{R_1(t)\cdot R_2(t)} = \frac{-\dot{R}_1(t)}{R_1(t)} + \frac{-\dot{R}_2}{R_2(t)} = z_1(t) + z_2(t)$$

Failure rates are added.

Interaction of different distributed failure effects (II)

Assumptions:

- Failures are independent,
- 2 failure effects, both **exponetial distributed** with λ_1 and λ_2.

$$R(t) = e^{-\lambda_1 t} \cdot e^{-\lambda_2 t} = e^{-(\lambda_1 + \lambda_2)t}$$

→ also exponential distributed with λ = λ1 + λ2

$$E = \frac{1}{\lambda_1 + \lambda_2}$$

Interaction of different distributed failure effects (III)

Assumptions:

- Failures are independent,
- 2 failure effects, both **weibull distributed** with α_1, β_1 and α_2, β_2.

$$R(t) = e^{-\frac{1}{\alpha_1}t^{\beta_1}} \cdot e^{-\frac{1}{\alpha_2}t^{\beta_2}} = e^{-\left(\frac{1}{\alpha_1}t^{\beta_1} + -\frac{1}{\alpha_2}t^{\beta_2}\right)}$$

→ only by β1 = β2 weibull distributed with $1/\alpha = 1/\alpha_1 + 1/\alpha_2$

$$z(t) = \frac{\beta_1}{\alpha_1}t^{\beta_1 - 1} + \frac{\beta_2}{\alpha_2}t^{\beta_2 - 1},$$

f(t)=z(t)*R(t)

$$E = ?$$