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Risk Analysis of Highly-integrated Systems

AM I : Network theory for the vulnerability analysis of infrastructure systems

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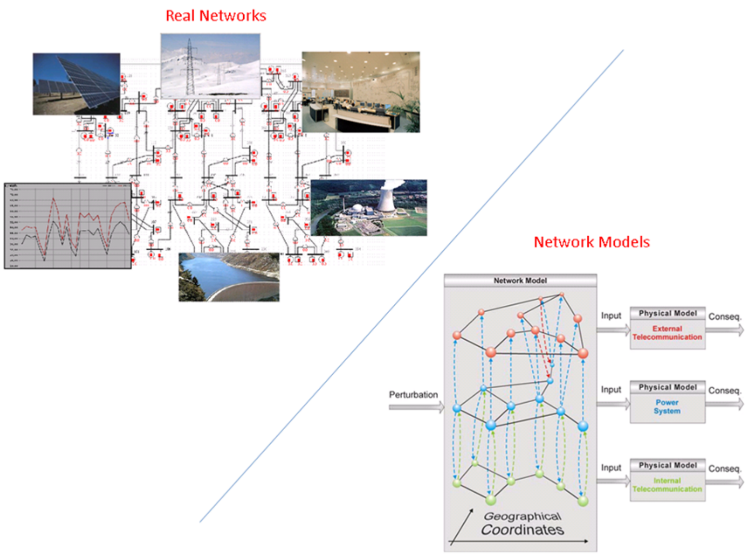

Introduction and Problem Description (I)

- Infrastructure systems provide essential goods and services to the industrialized society including transport, water, communication and energy.
- A disruption or malfunction often has a significant economic impact and potentially propagates to other systems due to mutual interdependencies.
- Wide-area breakdowns of such large-scale engineering networks are often caused by technical equipment failures and their coincidence in time which eventually result in a series of fast cascading component outages.
- Illustrative examples are a number of large electric power blackouts and near-misses as has been increasingly experienced in the last few years

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Introduction and Problem Description (II)

How can we quantify the reliability of infrastructures and **assess the risk** of such large-area breakdowns?

Basic problem: Infrastructures are **highly complex** and interdependent systems, consisting of an enormous number of technical and non-technical, interacting components; classic reliability analysis methods become limited due to the state space explosion.

Example: Consider a system of N=20 components with up state and down state. A “state enumeration approach”, such as a complete Markovian chain would have to consider *$2^N = 2^{20} \sim 10^6$ system states!*

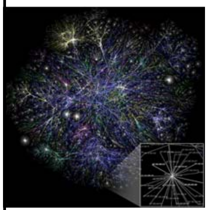
Approach 1: *Simulate the systems realistically by means of extensive modeling methods, including physical laws and operational dynamics.*

Introduction and Problem Description (III)

Approach 2: *Use highly simplified models in order to understand the* ***basic mechanisms*** *leading to infrastructure breakdowns. In this respect* ***network theory*** *allows for gaining valuable qualitative knowledge about the basic functioning of infrastructure systems, being networks in nature.*

However, due to its highly simplifying approach, network theory cannot replace more detailled reliability analysis methods.

It rather serves as a first „screening analysis“, whereas the findings, e.g. robustness of topology, may serve as an *input* for detailled reliability studies (and vice versa).



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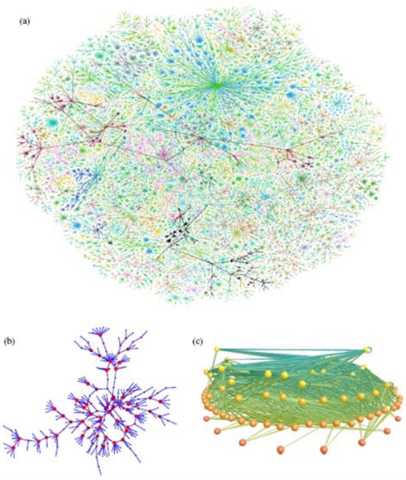
What can be represented as networks?

A few examples:



(a) Internet at the level of "autonomous systems"

(b) A social network (e.g. sexual contacts)

(c) A food web of predator-prey interactions between species.

Source: Newman, SIAM Rev. 45, 167 (2003).

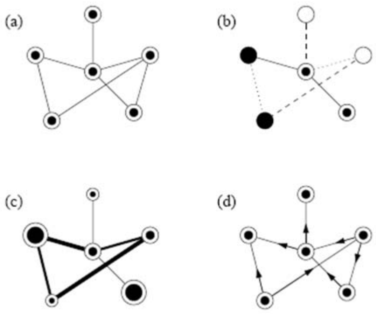
Network Characteristics: Some Basic Notations

- A network (or **graph**) is a set of *N* **nodes** (or **vertices** or **sites**) connected by *L* **links** (or **edges** or **bonds**)
- **G**(*N,L*): arbitrary graph of **order** *N* and **size** *L*
- Networks with undirected links are called **undirected networks** (a), those with directed links are called **directed networks** (b)

(a) (b)

- The total number of connections of a node to ist nearest neighboring nodes is called its **degree** *k*

Network Characteristics: Vertex Types and Edge Weights to Represent more Diversity



(a) an undirected network with only a single type of vertex and a single type of edge; (b) a network with a number of discrete vertex and edge types; (c) a network with varying vertex and edge weights; (d) a directed network in which each edge has a direction.
Source: Newman, SIAM Rev. 45, 167 (2003).

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Network Characteristics: Adjacency Matrix

- The adjacency matrix ***A*** provides a complete description of a network
- Consider a network with *N* nodes labelled by their index *i* (*i*=1,…,*N*). Then the adjacency matrix is a *N x N* matrix with elements a_{ij} :

if the network is undirected:

$a_{ij} = a_{ji}$, $a_{ij} = 1$ *if there exists a link between node i and j*

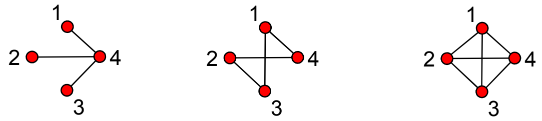
$a_{ij} = 0$ *otherwise*

if the network is directed:

$a_{ij} \neq a_{ji}$, $a_{ij} = 1$ *if there exists a link leaving node i and going to node j*

$a_{ij} = 0$ *otherwise*

- Examples for undirected graphs:



$$\begin{array}{c|cccc} & a_1 & a_2 & a_3 & a_4 \\ \hline a_1 & 0 & 0 & 0 & 1 \\ a_2 & 0 & 0 & 0 & 1 \\ a_3 & 0 & 0 & 0 & 1 \\ a_4 & 1 & 1 & 1 & 0 \end{array} \qquad \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \qquad k_i = \sum_j a_{ij}$$

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Network Characteristics: Degree distribution

The **degree distribution** *P(k)* gives the probability that any randomly chosen vertex has degree *k.*

Poisson

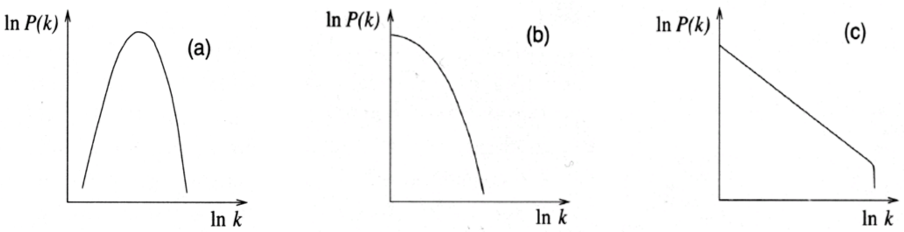
$$P(k) = \frac{e^{-\alpha}\alpha^k}{k!}, \ \text{where } \alpha = \overline{k}$$

Exponential

$$P(k) \propto e^{-k/\alpha}, \ \text{where } \alpha = \overline{k}$$

Power law

$$P(k) \propto k^{-\gamma}, \ k \neq 0$$



Source: Dorogovtsev, S. N. and Mendes (2003)

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Examples of typical degree distributions

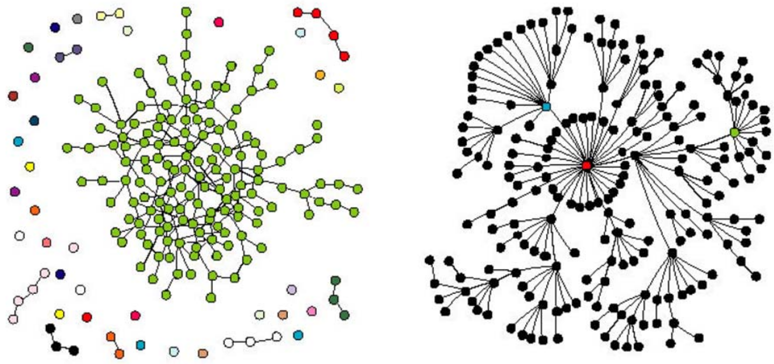

Illustration of network architectures. Left: random graph (Poisson), right: scale-free network (power law). Source: Strogatz, S. H., Nature 410, 268-276 (2001)

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Degree distributions – WWW

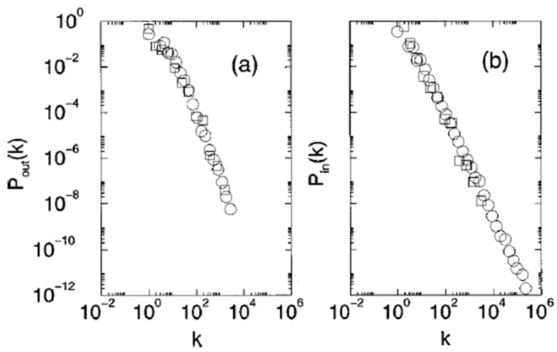

FIG. 2. Degree distribution of the World Wide Web from two different measurements: □, the 325 729-node sample of Albert *et al.* (1999); ○, the measurements of over 200 million pages by Broder *et al.* (2000); (a) degree distribution of the outgoing edges; (b) degree distribution of the incoming edges. The data have been binned logarithmically to reduce noise.

Source: Albert, R., Barabàsi, A.: Statistical Mechanics of Complex Networks, Rev. Mod. Phys., Vol. 74 (2002)

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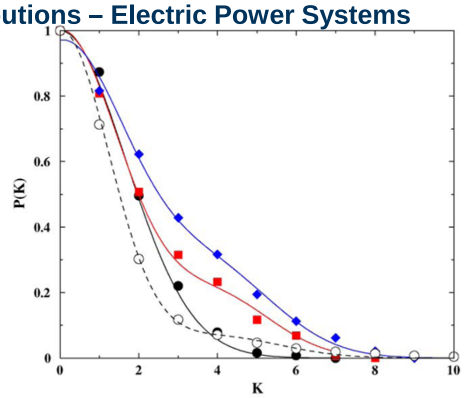
Degree distributions – Electric Power Systems



Cumulative distribution of the node degrees for the high-voltage transmission networks in Italy (full circles), Spain (diamonds) and France (squares). The empty circles represent the Italian „fine-grain“ network (from 380kV down to the distribution level).

Source: V. Rosato, *S. Bologna, F. Tiriticco*: Topological properties of high-voltage electrical transmission networks, Electric Power Systems Research, Vol. 77, 2007

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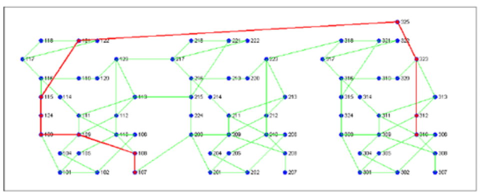
Network Characteristics: Shortest Path and Diameter

Shortest Path between node 107 and 310



Different *algorithms* are used to find the shortest path ℓ_{ij} between two nodes *i* and *j*, e.g. Dijkstra‘s algorithm

Average path length: average of all shortest paths in the network:

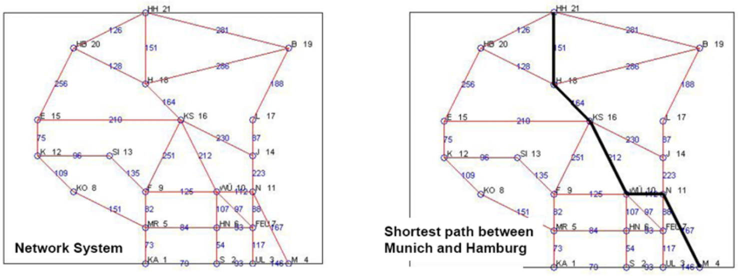
$$\langle \ell \rangle = \frac{1}{N(N-1)} \sum_{ij} \ell_{ij}.$$

Its value becomes infinity in case of a network splitting, due to e.g. disruption.

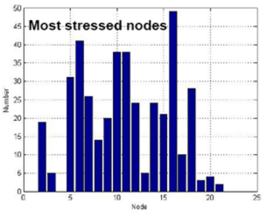
Network diameter: $d_G = \max_{i,j} \ell_{ij}$

Network Characteristics: Shortest Path (II)

Example: Highway Network



Most stressed nodes: most utilized nodes in all shortest paths



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Network Characteristics: Clustering Coefficient C

- How interlinked are my friends?

C measures the density of connections around a particular node.
Suppose you have *z* close friends.
If they all are again friends among themselves there will be:

$$C_{max} = \frac{z(z-1)}{2}$$

links between them. Suppose that there are only *y* connections between them. *C* will be

$$C = \frac{2y}{z(z-1)} = \frac{y}{C_{max}}$$

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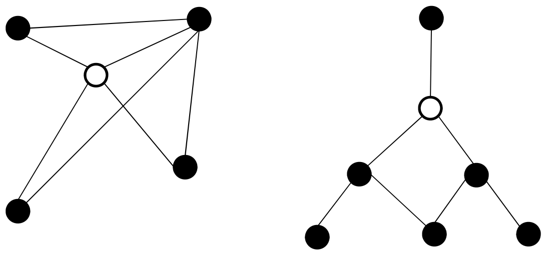
Network Characteristics: Clustering Coefficient *C* (II)



How would you calculate the clustering coefficient for the white node?

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Characteristics of Real-life Networks

	Network	Type	*N*	*L*	$\bar{k}$	ℓ	*γ*	*C*
Social	film actors	undirected	449 913	25 516 482	113.43	3.48	2.3	0.78
	company directors	undirected	7 673	55 392	14.44	4.60	–	0.88
	math coauthorship	undirected	253 339	496 489	3.92	7.57	–	0.34
	physics coauthorship	undirected	52 909	245 300	9.27	6.19	–	0.56
	biology coauthorship	undirected	1 520 251	11 803 064	15.53	4.92	–	0.60
	telephone call graph	undirected	47 000 000	80 000 000	3.16		2.1	
	email messages	directed	59 912	86 300	1.44	4.95	1.5/2.0	0.16
	email address books	directed	16 881	57 029	3.38	5.22	–	0.13
	student relationships	undirected	573	477	1.66	16.01	–	0.001
	sexual contacts	undirected	2 810				3.2	
Information	WWW nd.edu	directed	269 504	1 497 135	5.55	11.27	2.1/2.4	0.29
	WWW Altavista	directed	203 549 046	2 130 000 000	10.46	16.18	2.1/2.7	
	citation network	directed	783 339	6 716 198	8.57		3.0/–	
	Roget's Thesaurus	directed	1 022	5 103	4.99	4.87	–	0.15
	word co-occurrence	undirected	460 902	17 000 000	70.13		2.7	0.44
Technological	Internet	undirected	10 697	31 992	5.98	3.31	2.5	0.39
	power grid	undirected	4 941	6 594	2.67	18.99	–	0.080
	train routes	undirected	587	19 603	66.79	2.16	–	0.69
	software packages	directed	1 439	1 723	1.20	2.42	1.6/1.4	0.082
	software classes	directed	1 377	2 213	1.61	1.51	–	0.012
	electronic circuits	undirected	24 097	53 248	4.34	11.05	3.0	0.030
	peer-to-peer network	undirected	880	1 296	1.47	4.28	2.1	0.011
Biological	metabolic network	undirected	765	3 686	9.64	2.56	2.2	0.67
	protein interactions	undirected	2 115	2 240	2.12	6.80	2.4	0.071
	marine food web	directed	135	598	4.43	2.05	–	0.23
	freshwater food web	directed	92	997	10.84	1.90	–	0.48
	neural network	directed	307	2 359	7.68	3.97	–	0.28

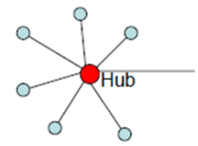
Exponent *γ* indicated only if it is a scale-free network

Source: Newman, SIAM Rev. 45, 167 (2003)

Random Failure and Attack Tolerance (I)

type of impact	exponential network	scale-free network
random	robust	extreme robust
malicious attack	robust	extreme vulnerable

scale-free network:



the chance to destroy the hub with a random attack is 1:7

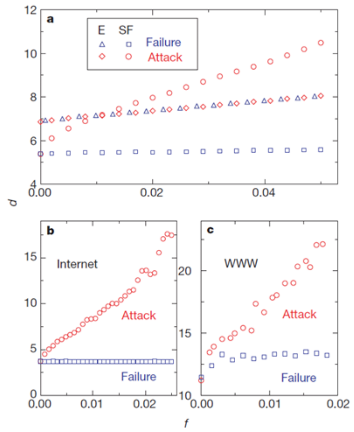
a malicious attack to the hub destroys the connection to six nodes

Random Failure and Attack Tolerance (II)

d: average length of the shortest paths between any two nodes in the network

f: fraction of the removed nodes

E: exponential
SF: scale-free



Albert, Jeong, Barabasi, Nature 406, 378, 2000

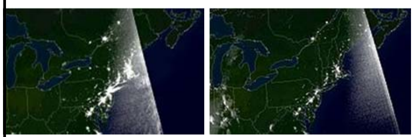
Cascading Failures in Infrastructure Networks

Picture: Flickr.com

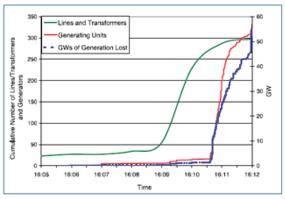
- are often the result of a relatively slow system degradation escalating into a fast avalanche of component failures, potentially leading to a complete loss of service
- while the first few outages might even be independent of each other, the causal failure chains usually become more pronounced in the course of the events, ending up in a fully cascading regime.

Example: The North American blackout 2003

The slow degradation started around noon with the outage of a system monitoring tool, further progressed during the afternoon through the independent outage of a generator and several transmission lines and finally evolved into the full cascade at around 16:00



Satellite image: day before and the night of the blackout



Component outages
(U.S.-Canada Power System Outage Task Force, 2004)

How to Analyze Cascading Failures?
A simple load redistribution model (Motter and Lai)[1]

Model:

- The **load *L*** at a node is the total number of shortest paths passing through the node.
- The **capacity *C*** of a node is the maximum load that the node can handle. In man-made networks, the capacity is limited by cost. Thus, it is assumed that the capacity C_j of node j is proportional to its initial load L_j:

$$C_j = (1+\alpha)L_j, \quad j = 1,2,...N$$

- The removal of nodes, in general, changes the distribution of shortest paths.
- The load at a particular node can then change; if it increases and becomes larger than the capacity C_j, the corresponding node fails.
- Any failure leads to a new redistribution of loads and, as a result, subsequent failures can occur ⟶ ***cascading failure***
- Measure for the size of a cascade:

$$G = N'/N$$

N and N' are the number of nodes in the largest component before and after the cascade, respectively.

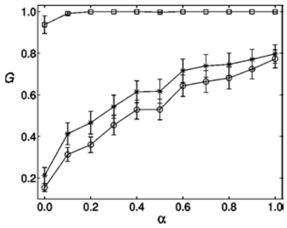
[1] Motter, A. E., Lai Y.-C., Cascade-based attacks on complex networks, Physical Review E 66, 065102

How to Analyze Cascading Failures?

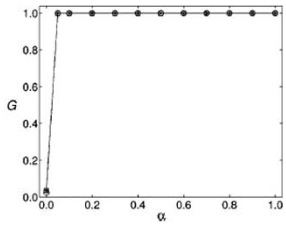
What can we learn from such a simple, abstract modeling approach?

global cascades occur if
1. the network exhibits a highly heterogeneous distribution of loads (i.e. heterogeneous networks such as scale-free networks)
2. the removed node is among those with higher loads.

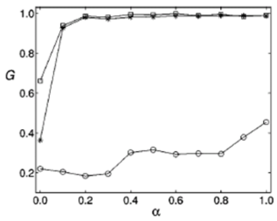
Otherwise, cascades are not expected.



scale-free networks (N~5000)



homogeneous networks (e.g. random graph) (N=5000)



western U.S. power transmission grid (N=4941)

The cascades are triggered by the removal of single nodes chosen at random (squares), or among those with largest degrees (asterisks) or highest loads (circles)

Recommended literature on network theory:

- Dorogovtsev, S. N. and Mendes, J. F. F., "Evolution of Networks - from Biological Nets to the Internet and WWW",(Oxford University Press, Oxford, 2003)
- Barrat, A. and Barthelemy, M. and Vespignani, A., "Dynamical processes on complex networks",
(Cambridge University Press, 2008)
- Newman, M., „The structure and function of complex networks", (SIAM Rev. 45, 167, 2003)