# Risk Analysis of Highly-integrated Systems

**Additional information on modeling and simulation tools**

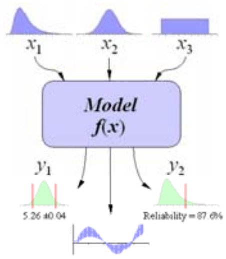
**(Monte Carlo methods, finite state modeling/state models, Markov chains, Petri nets)**



## Monte Carlo Simulation

Definition: The use of randomly generated data and computer simulations to obtain approximate solutions to complex mathematical and statistical problems.

Step 1: Create a parametric model, y = f(x1, x2, ..., xq).
Step 2: Generate a set of random inputs, xi1, xi2, ..., xiq.
Step 3: Evaluate the model and store the results as yi.
Step 4: Repeat steps 2 and 3 for i = 1 to n.
Step 5: Analyze the results using histograms,
summary statistics, confidence intervals, etc.




Monte Carlo is about invoking laws of large numbers to approximate expectations

## State Model

Instead of subdividing a system into components it can also be subdivided into **global states** $Z_1, \ldots, Z_m$. Each state $Z_i$ represents a combination of component states: $Z_i = (K_1, \ldots, K_n)$.

The **transition rates** $\alpha_{i,j}$ between states $Z_i$ and $Z_j$ define the mean number of transitions from $Z_i$ to $Z_j$ per time unit provided that the system is in state $Z_i$ ($1 < i < m$ und $1 < j < m$).

Advantages:

- Modeling in great detail: Any state transitions can be expressed, in particular: fault tolerance techniques (like reconfiguration) and **repair**.
- joint performance and reliability evaluation

Disadvantage: High computation overhead due to the large state space.

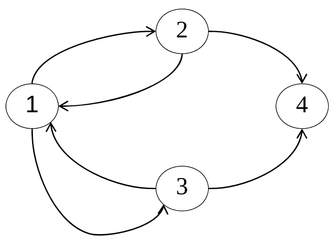
**$p_i$ is a probability of a state i.**





## Finite state machines

A model consisting of a set of states *S*, a start state, possible transitions and a transition function that maps states to a next state. It changes to new states depending on the transition function.




<u>Example</u>: 4 States
2 Components in Parallel

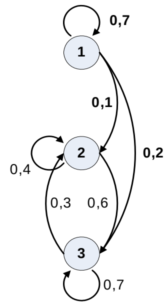
State 1: Both components working
(start state)
State 2: Component 1 failed
State 3: Component 2 failed
State 4: Component 1 and 2 failed
Final state (no transitions out)

## Markov Chains

A Markov chain consists of a state space *S={1,...,n}* and a transition Matrix *T* that defines the probabilities of each transition.

Transition Matrix

$$\begin{pmatrix} 0{,}7 & 0{,}1 & 0{,}2 \\ 0 & 0{,}4 & 0{,}6 \\ 0 & 0{,}3 & 0{,}7 \end{pmatrix}$$



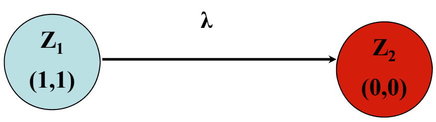

Markov chains are directed graphs that have a weight (numeric value) associated with each edge





**Application of the State Modell:**

**Unrepairable System**




**$Z_i(K_i,S)$, where $K_i$ state of component, S- conditional state of system.**

**Λ - failure rate**

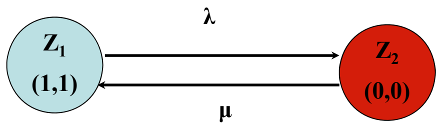
**Homogenous system of equalities:** $$\left(\dot{p}_1(t), \dot{p}_2(t)\right) = \left(p_1(t), p_2(t)\right) \cdot \begin{pmatrix} -\lambda & \lambda \\ 0 & 0 \end{pmatrix}$$

**To be solved unter conditions: $p_1(t)+p_2(t)=1$ and $p_1(0)=1$**

**Solution (via Laplace transformation):** $p_1(t) = e^{-\lambda \cdot t}$ *and* $p_2(t) = 1 - e^{-\lambda \cdot t}$

## Application of the State Modell:

### Repairable System




**$Z_i(K_i,S)$, where $K_i$ state of component, S- conditional state of system.**

**Λ - failure rate, μ – repair rate**

**Homogenous system of equalities:** $$\left(\dot{p}_1(t), \dot{p}_2(t)\right) = \left(p_1(t), p_2(t)\right) \cdot \begin{pmatrix} -\lambda & \lambda \\ \mu & -\mu \end{pmatrix}$$

**To be solved unter conditions: $p_1(t)+p_2(t)=1$ and $p_1(0)=1$**

**Solution:** $$p_1(t) = \frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu} \cdot e^{-(\lambda+\mu)\cdot t}$$

**It is the time dependent Availability V(t)!**





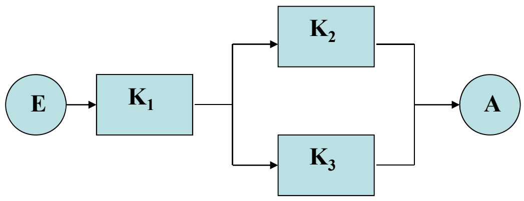
State models express each combination of component states by a separate global state.

In a system from two components $K_1$ and $K_2$ we have the global states:
- Both $K_1$ and $K_2$ are faultless, expressed by the global state $Z_1$ = (1, 1).
- $K_1$ is faultless and $K_2$ is faulty, expressed by the global state $Z_2$ = (1, 0).
- $K_1$ is faulty and $K_2$ is faultless, expressed by the global state $Z_3$ = (0, 1).
- Both $K_1$ and $K_2$ are faulty, expressed by the global state $Z_4$ = (0, 0).

The transition rate $\alpha_{i,j}$ can also be seen as the reciprocal of the mean duration between the entering of state $Z_i$ and the transition from $Z_i$ to $Z_j$. However, it must be taken into account, that from $Z_i$ also other states could be reached.

## Application of the State Modell:

### Example System

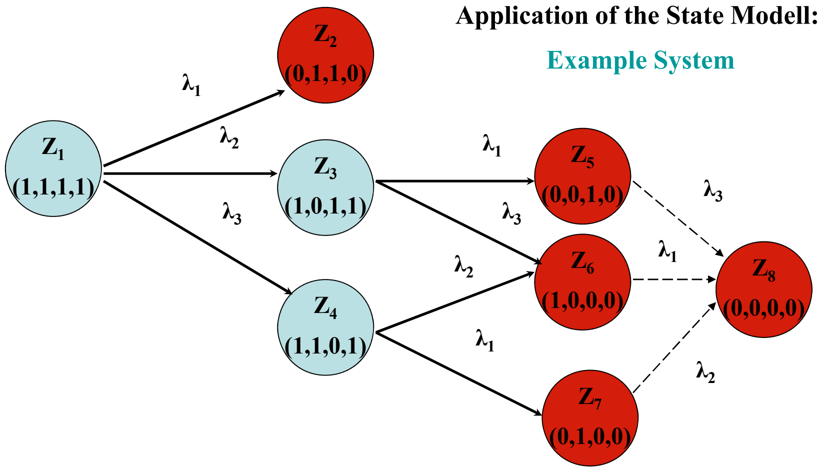



Number of states $2^3=8$



## Application of the State Modell:

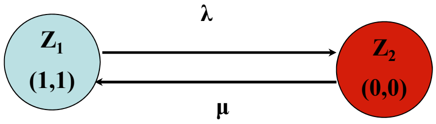
### Example System




Stationary availability is calculated by $V=p_1+p_3+p_4$

**Application of the State Modell:**

**Repairable System**




$Z_1$: $\lambda * p_1 = \mu * p_2$

$Z_2$: $\lambda * p_1 = \mu * p_2$

$p_1 + p_2 = 1$

**Stationary availability (steady-state availability)**

**V is calculated by**

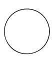
$$p_1 = \frac{\mu}{\lambda + \mu}, \text{ the complement is } p_2 = \frac{\lambda}{\lambda + \mu}$$





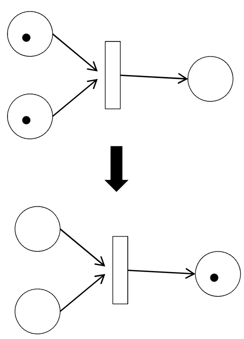
## Petri Nets

The Petri net is a directed graph with nodes representing either "places" (represented by circles) or "transitions" (represented by rectangles). When all the places with arcs to a transition (its input places) have a token, the transition "fires", removing a token from each input place and adding a token to each place pointed to by the transition (its output places).

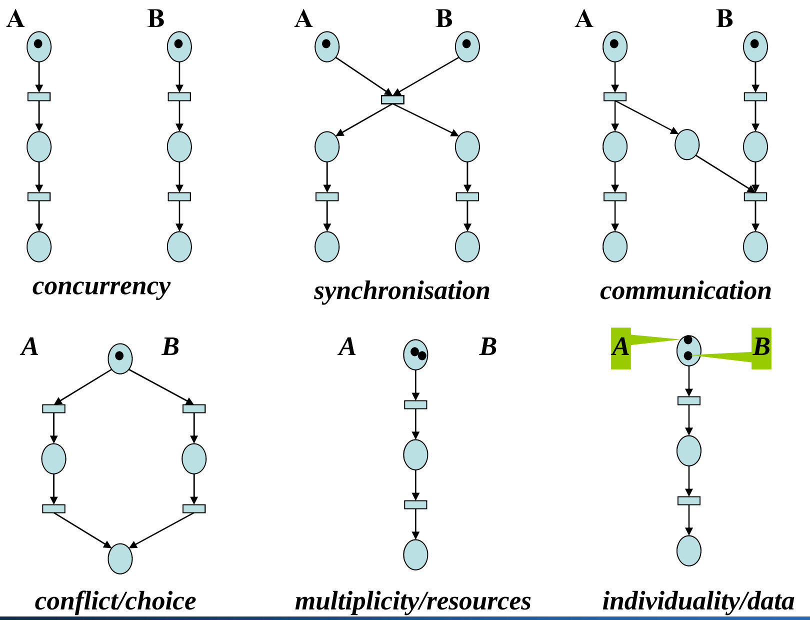
 Places (Storage)

 Transitions (Actions)

 Edges



## Modelling








## Examples of technical Networks

Swiss Power System


Natural Gas Pipelines

- Internet
- World Wide Web
- Railway
- Motorway
- …

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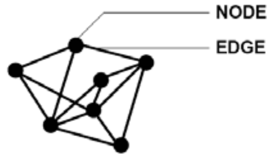
## Definition: Graph

Formal definition:

Tuple $(V,E)$ with $V$ a set of $n$ vertices and a set of $m$ edges $E$: $\mathbf{G} = (V,E)$




Node Set V (G)
Edge set E={(a,b)}



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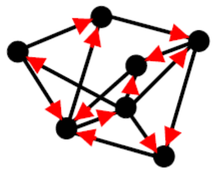
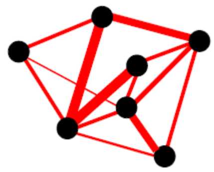
## Types of Graphs

- Directed graphs
- Weighted graphs



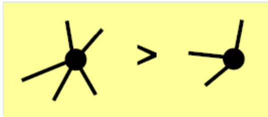
Directed graphs



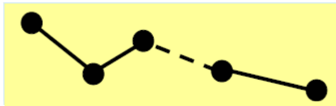
Weighted Graph

## Definitions: Degree and connectedness

The degree of a vertex v in graph G is
$d_G(v) = |N_G(v)|$.




G is connected, if there is a path u between two vertices.



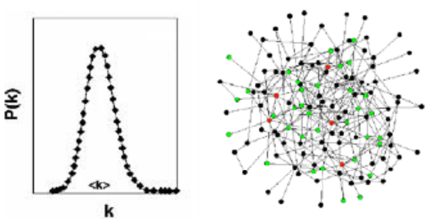
Otherwise G is disconnected.



## Types of Networks

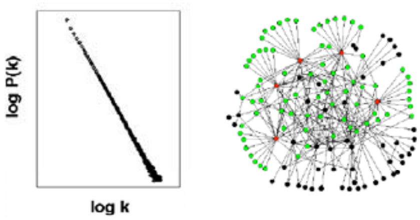
Random network

Poisson distribution of the number k of edges between nodes




Scale-free network

Exponential distribution of the number of edges k between nodes




**Examples: Internet, Power System**

**Figures: Bornholdt, Schuster: Handbook of Graphs and Networks**

## Vulnerability Assessment of Networks
## Real Networks

Most of the technical networks are scale-free.

e.g. Power System, Internet

Reasons:

- less expensive, fewer edges necessary
(end user needs only one connection)
- efficient
- natural growth



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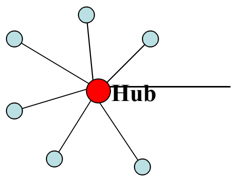
## Vulnerability of Network Types

| type of impact | exponential network | scale-free network |
|---|---|---|
| random | robust | extremely robust |
| malicious attack | robust | extremely vulnerable |

scale-free network:




the chance to destroy the hub with a random attack is 1:7

a malicious attack to the hub destroys the connection to six nodes

# Vulnerability Assessment of Networks
## Network Parameters

Measures to characterize and analyze the vulnerability of a network with N vertices and M edges

Size of the graph: number of edges in the graph

Degree of distribution $k_i$: number of edges connecting vertex i; the average degree is given by k=2M/N

Clustering coefficient: ratio of existing and maximum possible number of edges between the neighboring vertices $k_i$ of a vertex i; neighboring vertices are the vertices actually connected to node i

Shortest path: shortest path between two vertices

Average path length: average of all shortest paths in the network

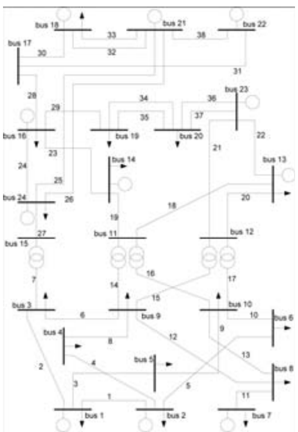
Most stressed edge: most utilized edge in all shortest paths



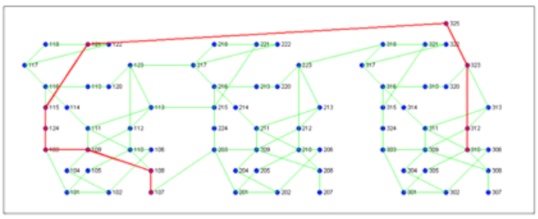
# Vulnerability Assessment of Networks – Shortest Path

**Dijkstra-Algorithm – one method to calculate the shortest path between two nodes:**

**IEEE-Test-System**

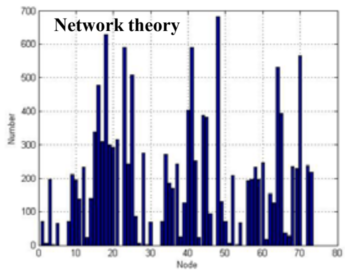


**Shortest Path between node 107 and 310**

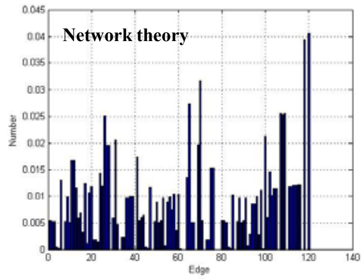


# Vulnerability Assessment of Networks – Most stressed edges and nodes

Most stressed node




Most stressed line





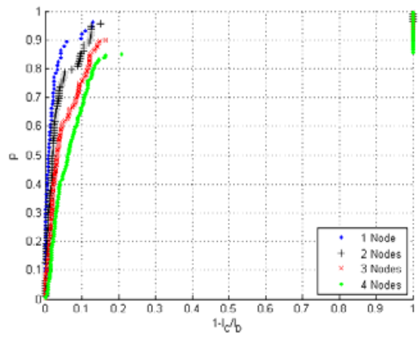


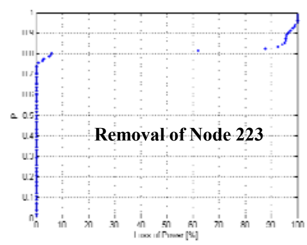

# Vulnerability Assessment of Networks – Removal of nodes

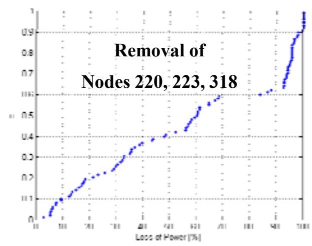
ABM – calculations

max loss of power

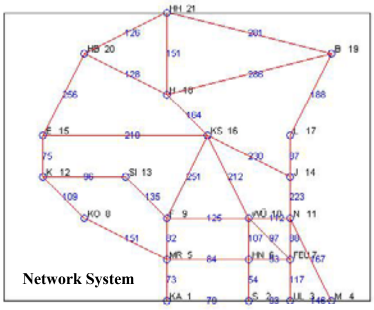
Network theory - Increase of the average path length after the removal of nodes

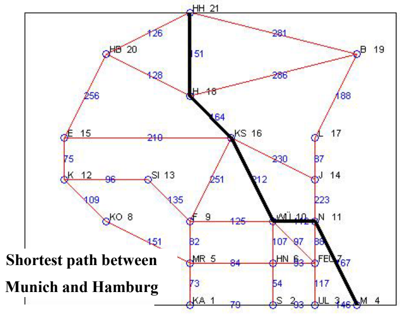


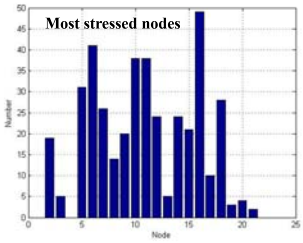



# Vulnerability Assessment of Networks – Highway Network



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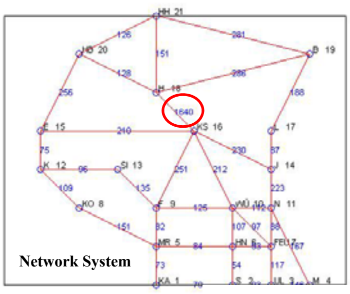


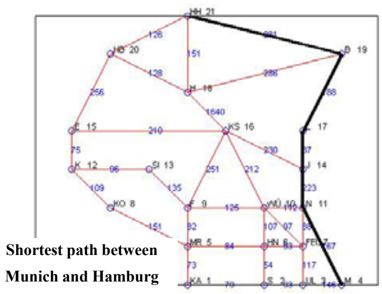


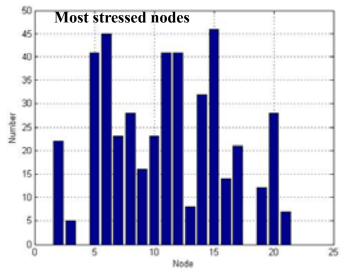

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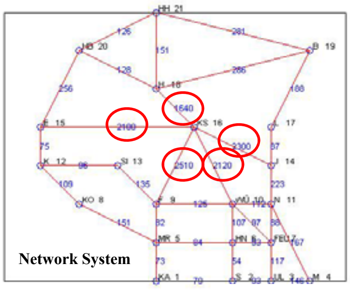


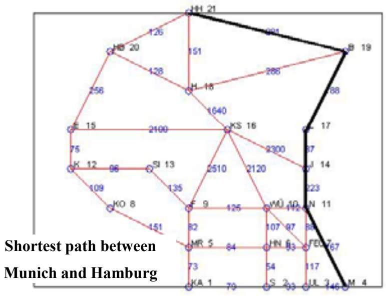


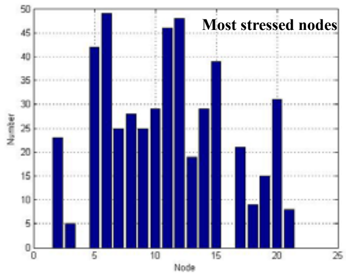






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