Cascading Disaster Spreading and Optimal, Network-Dependent Response Strategies

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with Lubos Buzna, Limor Issacharoff, Ingve Simonsen,
Christian Kühnert, Hendrik Ammoser, Karsten Peters
1) Causal dependencies and interaction networks

2) Modelling the spreading of failures

3) Recovery from disaster spreading

4) Examples: Power and freeway networks
Failure of Critical Infrastructures

Blackout Northern America, 2003: total loss of 6.7 billion USD, 50 Mio. people without electric power for about 24 hours.
Blackout Italy, 2003: total loss of 151 Mio. USD

Blackout in parts of the USA and Canada (2003), an impressing example of the long-reaching accompaniments of supply network failures.
Interaction Networks Behind Disaster Spreading

Example: Blackout USA 2003

- service disruptions of cellular phones
- wired phone network busy
- traffic lights off
- gas stations out of order
- public and private transport down
- oil refineries shut down
- advisory to boil water
- no electricity to boil water
- pumps without power
- wired phone network busy
Common Elements of Disasters
Causality Network for Thunderstorms

- **direct impact on environment, organisms**: thunderstorm heavy rainfall, hail, wind, etc.
- **direct impact on anthropogenic systems (material damage)**
  - materials damaged or destroyed
  - damaged or destroyed buildings or facilities
  - traffic accidents causing higher risk of accidents
  - temporary breakdown of electricity or water supply
  - breakdown of telecommunication infrastructure
  - emergency services overwhelmed
  - public life breaks down (administration, schools, industry, etc.)
  - proclamation of the state of emergency, declaration of disaster zones

- **economic impact**:
  - employment of affected people, tourism, etc.
  - economic impact on economy, infrastructure
  - flooding damages
  - impact on agriculture
  - food prices

- **impact on health and human bodies**
  - injured or dead persons
  - trauma in affected people
  - media interest, reports, and broadcasting
  - diseases, epidemics
  - hygienic problems
  - medical care

- **impact on society**
  - soldiers, homeless, etc.
  - evacuation of coastal areas
  - destruction and endanger of all traffic, delay, route diversions
  - emergency accommodations
  - emergency aid
  - insurance costs

- **indirect effects**
  - extreme events
  - flooding, inundation
  - power supply outage, outage of several electric powered facilities
  - power supply shortage
  - internet connections lost
  - drinking water supply

Disasters Cause Disasters

Causality Network of the Elbe Flooding 2002 (Detail)

- Recovery
- Communication
- Information
- Energy Supply
- Water level
- Drinking Water
- Public Transport
- Food Supply
- Medical Forces
- Health System

Arrows indicate the causality with positive (+) and negative (-) values.
Quantitative Analysis of Causality Networks

Identify the elements of the matrix $M$. Consider quantitative (data) and qualitative interactions \{\{-3, \ldots, +3\}\} and thus functional and structural characteristics of the causal networks for different means of disaster!
Modeling and Simulation of Disaster Spreading

Simulation of topology dependent spreading:

- What are the influences of different network topologies and system parameters?
- Optimal recovery strategies?

Buzna L., Peters K., Helbing D., Modelling the Dynamics of Disaster Spreading in Networks, Physica A, 2006

Spreading of disasters:
Causal dependencies (directed)
Initial event (internal, external)
Redistribution of loads
Delays in propagation
Capacities of nodes (robustness)
Cascade of failures

Scope of research:
Spreading conditions (network topologies, system parameters)
Optimal recovery strategies
Mathematical Model of Disaster Spreading

Node dynamics:

\[ \frac{dx_i}{dt} = -\frac{x_i}{\tau} + \Theta \left( \sum_{j \neq i} \frac{M_{ij}x_j(t - t_{ij}) e^{-\beta t_{ij}/\tau}}{f(O_i)} \right) + \xi_i(t) \]

- \( x_i \): state of the node
- \( x_i = 0 \): usual situation
- \( x_i > \theta_i \): node is destroyed

\[ \Theta(x) = \frac{1 - \exp(-\alpha x)}{1 + \exp[-\alpha(x - \theta_i)]} \]

\[ f(O_i) = \frac{aO_i}{1 + bO_i} \]

- \( \theta_i \): node threshold
- \( 1/\tau \): healing rate
- \( t_{ij} \): time delay
- \( \xi_i(t) \): internal noise
- \( M_{ij} \): link strength
- \( O_i \): node out-degree
- \( a, b, \alpha, \beta \): fit parameters

Threshold function:

\[ \Theta(x) = \frac{1 - \exp(-\alpha x)}{1 + \exp[-\alpha(x - \theta_i)]} \]

Node degree:

\[ f(O_i) = \frac{aO_i}{1 + bO_i} \]

We use a directed network, dynamical, bistable node models and delayed interactions along links.
Failures Triggered by Internal Fluctuations

Coinciding, distributed, random failures:

\[
\frac{dx_i}{dt} = -\frac{x_i}{\tau} + \Theta \left( \sum_{j \neq i} \frac{M_{ij}x_j(t - t_{ij})}{f(O_i)} e^{-\beta t_{ij}/\tau} \right) + \xi_i(t)
\]

Damage compared to an “unconnected network”:


Connectivity is an important factor (in a certain region).
Phase Transition in Disaster Spreading

Node robustness vs. failure propagation:

\[
\frac{dx_i}{dt} = -\frac{x_i}{\tau} + \Theta \left( \sum_{j \neq i} \frac{M_{ij} x_j(t - t_{ij})}{f(O_i)} e^{-\beta t_{ij}/\tau} \right) + \xi_i(t)
\]

We found a critical threshold for the spreading of disasters in networks. Topology and parameters are crucial.
We found a topology dependent „velocity“ of failure propagation. Spreading in scale-free networks is slow.

Modelling the Recovery of Networks

1. Mobilization of external resources:

\[ r(t) = a_1 t^{b_1} e^{-c_1 t} \]

2. Formulation of recovery strategies as a function of
- the network topology
- the level of damage

\[ \frac{1}{\tau_i(t)} = \frac{1}{(\tau_{\text{start}} - \beta_2) e^{-\alpha_2 R_i(t)} + \beta_2} \]

3. Application of resources in nodes

Parameters:
- \( t_D \) time delay in response
- \( R \) disposition of resources

\( R_i(t) \) cumulative number of resources deployed at node \( i \)
\( \tau_{\text{start}} \) initial intensity of recovery process
\( \alpha_2, \beta_2 \) fit parameters
Mobilization of Resources

Example: Mobilization during the Elbe flood 2002:

Mobilization of resources (time dependent)

External resources become available after a certain response time delay $T_D$

During mobilization the number of resources increases

Later a phase of demobilization occurs

Number of available resources $r(t)$:

$$R(t) = a_1 t^{b_1} e^{-c_1 t}$$

$a_1$, $b_1$, $c_1$ are fit parameters
Recovery Strategies

Application of external resources in nodes:

\[ \tau(t) = (\tau_{\text{start}} - \beta) e^{\alpha R_i(t)} \]

- \( R_i(t) \): cumulative number of resources deployed at node \( i \)
- \( \tau_{\text{start}} \): time to start healing
- \( \alpha, \beta \): fit parameters

Formulation of recovery strategies as a function of the
- network topology
- level of damage

\( S_0 \) – no recovery
\( S_1 \) – uniform deployment
\( S_2 \) – priority 1: destroyed nodes
priority 2: damaged nodes
\( S_3 \) – out-degree based deployment
How to Distribute Available Resources?

Formulation of recovery strategies, based on information:

- **S₀**: no recovery

**Topology information only:**
- **S₁**: uniform deployment
- **S₂**: out degree based dissemination

**Damage information:**
- **S₃**: uniform reinforcement of challenged nodes $(x_i > 0)$
- **S₄**: uniform reinforcement of destroyed nodes $(x_i > 0)$

**Damage & topology information:**
- **S₅**: targeted reinforcement of highly connected nodes
  1st priority: fraction $q$ to hub nodes
  2nd priority: fraction $1-q$ according to $S₄$
- **S₆**: out-degree based targeted reinforcement of destroyed nodes

Application of resources to a scale-free network

Start of recovery

- $S₀$
- $S₄$
- $S₃$
- $S₅$
- $S₆$

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Recovery of Networks

Parameters:  
Network topology  
time delay in response  \( t_D = 8 \)  
disposition of resources  \( R = 1000 \)

L. Buzna, K. Peters, H. Ammoser, Ch. Kuehnert and D. Helbing:  
Efficient response to cascading disaster spreading,  
Behaviour of a Node for Sufficient and Insufficient Resources

Minimum Quantity of Resources $R_{\text{min}}$ for Recovery

Given: Amount of resources, mobilized with certain delay.

Is the network able to recover?

Minimum quantity of resources needed to recover a challenged network as a function of the response time delay and network topology.

- Recovery (in reasonable time) is not always possible.
Recovery of Networks: When Does Strategy Matter?

Comparison of efficient and inefficient strategies:

- The delay of recovery activities is crucial.

- Optimization of recovery strategies is promising in certain parameter regions.
Comparison of Efficient and Inefficient Strategies

Relative difference in damage between $S_6$ and $S_1$

$$D_{6,1} = (\langle D_6 \rangle / \langle D_1 \rangle) \times 100\%$$

1. $D_{6,1} = 80\%$
2. $D_{6,1} = 20\%$

- $S_1$ - uniform dissemination (the worst strategy)
- $S_6$ – out – degree based targeted reinforcement of destroyed nodes (the best strategy)

1. The promptness of recovery activities has a crucial influence on their efficiency
2. Optimization of protection strategies is possible in certain parameter regions
Most Efficient Strategies

There is no unique optimal response strategy:

1. Strategies based on the network structure has been proved as a most suitable for scale-free structures.
2. Strategies based on the damage information are more appropriate for regular networks.
3. The situation in Erdős-Rényi and small-world networks depends on $t_D$ (short $t_D \Rightarrow$ damage based strategies)  
   (large $t_D \Rightarrow$ network structure based strategies)
Mixed Recovery Strategies

Objectives:
- Minimal average damage
- Minimal sufficient quantity of resources

Parameters:
- $R$ overall disposition of resources
- $t_D$ time delay of recovery
- Network topology

Methods:
- Mixing of basic strategies
- Switching between strategies in time

Application of resources ($R = 2000$) on scale-free network

Destroyed nodes vs. Time

$S_4$: 57% $S_2$ 43% $S_3$
Network-Dependence of Best Strategy

Strategies based on the network structure have been proven most suitable for scale-free structures.

Strategies based on information about the degree of damage are more appropriate for regular networks.

The situation in Erdös-Rényi and small-world networks depends on the response time $t_D$
(short $t_D$ $\Rightarrow$ orient at damage)
(large $t_D$ $\Rightarrow$ orient at network structure)
Critical Infrastructures and Their Vulnerability

- "Critical Infrastructures consist of those physical and information technology facilities, network services and assets which, if disrupted or destroyed, would have a serious impact on the health, safety, security or economic well-being of citizens or the effective functioning of governments". (Commission of the European Communities in 2004)

- A system is said to be **vulnerable** if its functioning can be significantly reduced by intentional or non-intentional means.

\[
L = \frac{df}{du}
\]

- **Level of Vulnerability**
- **System’s functioning**
- **System’s failure**
In the case of freeways (no choice of different travel modes/means of transport), the classical 4-step model reduces to the following 3 steps:

1. Trip generation (overall traffic volume generated per hour)

2. Trip distribution (OD choice with multinominal logit model, exponentially distributed as function of travel time)

3. Traffic assignment (based on travel time, distribution over alternative routes according to the Wardrop principle)

Travel time on link \( l \) is modeled by the classical capacity constraint function:

\[
T_l(q_l) = T_l^0 \left[ a \left( 1 + \left( \frac{q_l}{k_l} \right)^b \right) \right]
\]
Topological Analysis

**Efficiency**

\[ E[G] = \frac{1}{N(N-1)} \sum_{i,j \in G} \frac{1}{d_{ij}} \]

\( d_{ij} \) – shortest path between nodes i and j

**Edge Information Centrality**

\( E(G) \) and \( E(G') \) is the efficiency before and after the links’ removal, respectively

\[ IC_{ij} = \frac{\Delta E}{E} = \frac{E[G] - E[G']}{E[G]} \]

**Edge Betweenness Centrality**

\( n_{ij} \) number of shortest paths between city nodes which pass through the edge connecting nodes i and j

\[ b_{ij} = \frac{n_{ij}}{(N-1)(N-2)} \]
Case Study: The Italian German, and French Highways

For each country we chose a subset of the network that includes the highways connecting 29 of the most populated cities.

<table>
<thead>
<tr>
<th></th>
<th>Italy</th>
<th>Germany</th>
<th>France</th>
</tr>
</thead>
<tbody>
<tr>
<td>population</td>
<td>10701491</td>
<td>17366502</td>
<td>8381434</td>
</tr>
<tr>
<td>nodes</td>
<td>43</td>
<td>43</td>
<td>52</td>
</tr>
<tr>
<td>road’s number</td>
<td>57</td>
<td>65</td>
<td>71</td>
</tr>
<tr>
<td>total road’s length (km)</td>
<td>4946</td>
<td>5453</td>
<td>8194</td>
</tr>
<tr>
<td>average road’s length (km)</td>
<td>86</td>
<td>83</td>
<td>115</td>
</tr>
<tr>
<td>$&lt; \text{degree} &gt;$</td>
<td>2.6</td>
<td>3.0</td>
<td>2.7</td>
</tr>
<tr>
<td>$&lt; b_{ij} &gt;$</td>
<td>0.0451</td>
<td>0.0328</td>
<td>0.0332</td>
</tr>
</tbody>
</table>
Topological Analysis

Edge Betweenness Centrality

Italy

Germany

France

Edge Information Centrality

Italy

Germany

France
Functional Vulnerability of a Freeway System

Topological efficiency

\[ E[G] = \frac{1}{N(N-1)} \sum_{i,j \in G} \frac{1}{d_{ij}} \]

\( d_{ij} \) – shortest path between nodes \( i \) and \( j \)

Flow related analysis

Efficiency

\[ E_F = \frac{1}{N(N-1)} \sum_{i,j \in OD} \frac{1}{C_{ij}} \]

Cost Function

\[ C_{ij} = q_{ij} T_{ij} \]

Quality of service

\[ QoS = \frac{E_F}{E_{F_{max}}} = \frac{\sum_{ij} C_{ij}^{-1}}{\sum_{ij} (C_{ij}^{\text{min}})^{-1}} \]

\[ L_F^{(u)} = 1 - \left\langle \frac{QoS^{(u)}}{QoS(0)} \right\rangle \]
Topological Vulnerability and Flow-Related Vulnerability

\[ L_T^{(u)} = \frac{\Delta E}{E} = \frac{E[G(0)] - E[G(u)]}{E[G(0)]} \]

\[ L_F^{(u)} = 1 - \left\langle \frac{QoS^{(u)}}{QoS(0)} \right\rangle \]

\( \lambda \) is the number of links simultaneously removed from the network
Blackouts and Cascading Effects in Electricity Networks

New York, August 14, 2003

Rome, September 28, 2003
Blackouts and Cascading Effects in Electricity Networks

State of the power grid shortly before the incident

Sequence of events on November 4, 2006

1, 3, 4, 5 – lines switched off for construction work
2 – line switched off for the transfer of a ship by Meyer -Werft

E.ON Netz’s report on the system incident of November 4, 2006, E.ON Netz GmbH
Blackouts and Cascading Effects in Electricity Networks

Failure in the continental European electricity grid on November 4, 2006

Dynamic Model of Cascading Failures

Network:

\[ \mathcal{N} \] set of nodes
\[ \mathcal{L} \] set of links
\[ \mathbf{W} \] adjacency matrix \((W_{ij} \geq 0, \text{ link weight})\)

Model dynamics:

\[
n_i(t + 1) = \sum_{j=1}^{\mathcal{N}} T_{ij} n_j(t) + n_i^{\pm}
\]

- \(n_i(t)\) number of particles hosted by node \(i\) at the time \(t\)
- \(T_{ij} = W_{ij} / w_j, \quad w_j = \sum_{\ell=1}^{\mathcal{N}} W_{\ell j}\)
- \(n_i^{\pm} > 0\) node is source, \(n_i^{\pm} < 0\) node is sink

Stationary and Dynamic Models of Cascading Failures

Model normalization:
\[ \rho_i(t) = n_i(t)/N \] nodal particle density
\[ c_i(t) = \rho_i(t)/w_i \] utilization of outflow capacity
\[ j^\pm_i = n_i^\pm/(Nw_i) \] sinks and sources term

Dynamic model:
\[ c(t + 1) = T c(t) + j^\pm \]

Stationary model:
\[ c(\infty) = c^{(0)}(\infty) + (1 - T)^+ j^\pm \]

\((1 - T)^+\) generalized inverse of matrix \(1 - T\)

Link flow:
\[ C_{ij}(t) = W_{ij} c_j(t) \] current on link from \(i\) to \(j\)
\[ L_{ij}(t) = C_{ij}(t) + C_{ji}(t) \]
Stationary and Dynamic Models for Cascading Failures

Initial failure

Stationary model

Dynamic model

$t = 1$

$t = 2$
Model Dynamics

Power grid simulation model

Our model

http://www.eurostaq.epfl.ch/users_club/newsletter/nl8.html

R. Sadikovic: Power flow control with UPFC, (internal report)
Model Dynamics

Power grid simulation model

Our model

R. Sadikovic: Use of FACTS devices for power flow control and damping of oscillations in power systems, 2006, PhD thesis, ETH Zurich
Model Dynamics

UK high voltage power grid topology (300-400 kV)
Stationary Model vs. Dynamic Model

Link capacities:
\[ C_{ij} = (1 + \alpha) \cdot L_{ij}, \]

- \( |\mathcal{N}| \) number of nodes
- \( |\mathcal{L}| \) number of links
- \( |\mathcal{N}_R| \) number of remaining nodes
- \( |\mathcal{L}_R| \) number of remaining links

\[ G_{\mathcal{L}}(\alpha) = \frac{|\mathcal{L}_R|}{|\mathcal{L}|} \approx G_{\mathcal{N}}(\alpha) = \frac{|\mathcal{N}_R|}{|\mathcal{N}|} = G(\alpha) \]
Conclusions

- We have developed models to represent causal interrelationships triggering cascading disaster spreading, allowing to compare the effectiveness of alternative response strategies.

- A time-dependent model of disaster spreading allowed us to describe the impact of the topology of interrelationship networks on the spreading dynamics.

- The efficiency of different disaster response/relief strategies could be tested by the same model. Different networks require different response strategies! A quick response is crucial.

- Another model has been used to evaluate the vulnerability of freeway networks in different European countries.

- A model of cascading failures in power grids showed that stationary spreading models underestimate the robustness of electrical power supply networks by 80% and more.