ETH
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Cascading Disaster Spreading and Optimal, Network-Dependent Response Strategies

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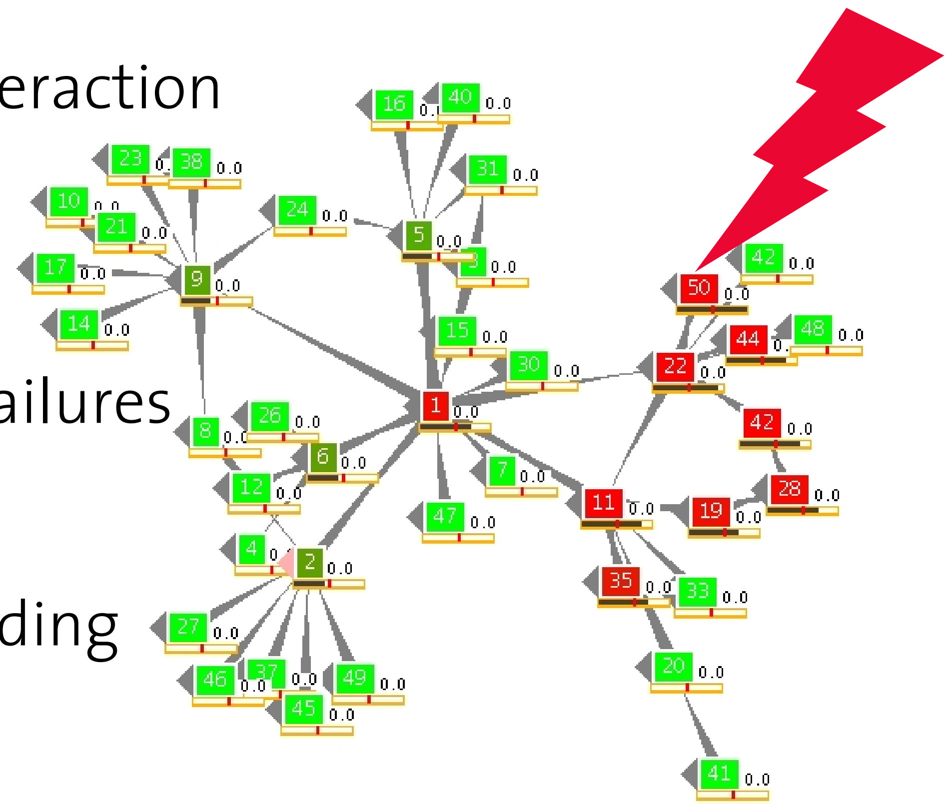
Disaster Spreading in Networks

1) Causal dependencies and interaction networks

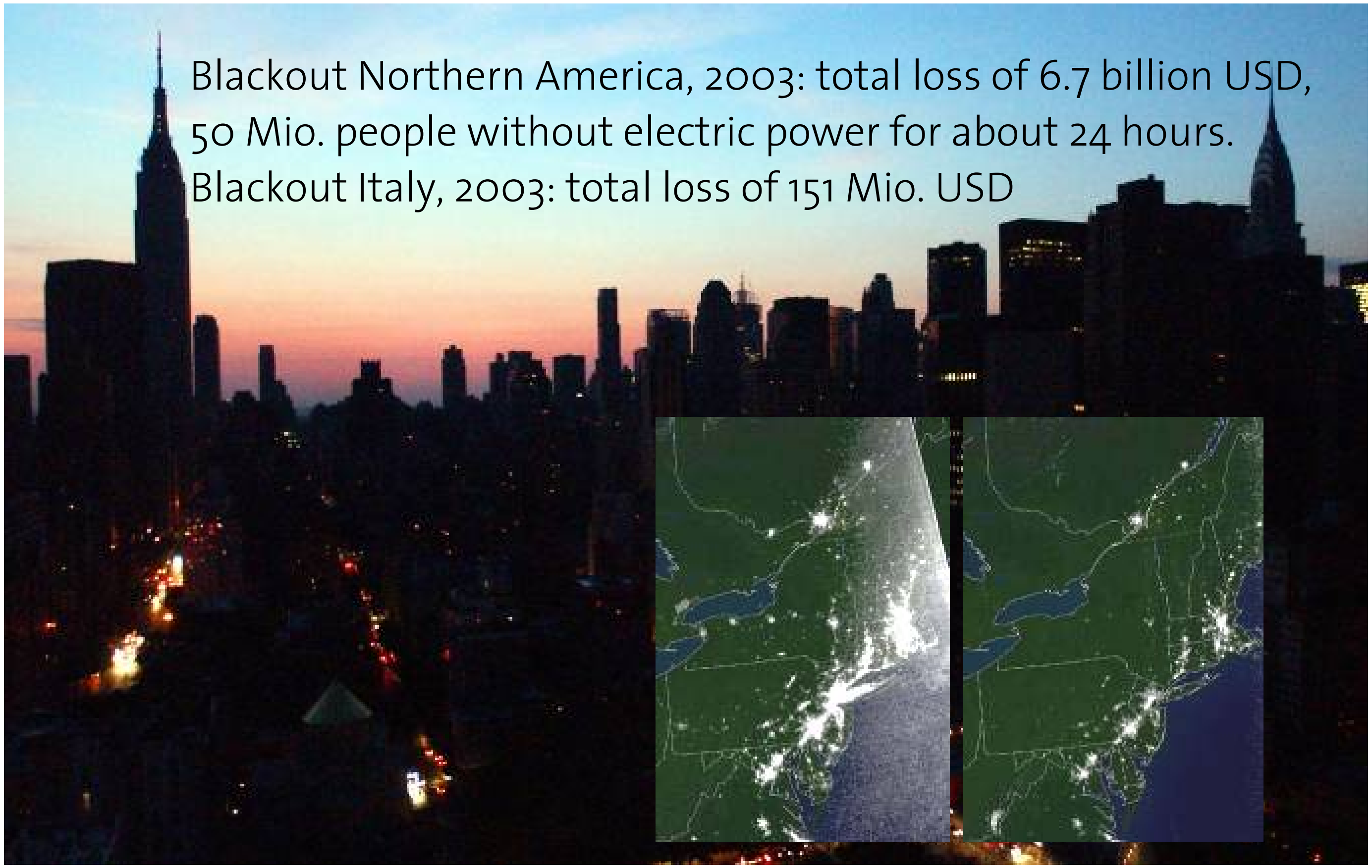
2) Modelling the spreading of failures

3) Recovery from disaster spreading

4) Examples: Power and freeway networks



Failure of Critical Infrastructures



Blackout in parts of the USA and Canada (2003), an impressing example of the long-reaching accompainments of supply network failures.

Interaction Networks Behind Disaster Spreading

Example: Blackout USA 2003

service disruptions of cellular phones

traffic lights off

wired phone network busy

gas stations out of order

blackout

public and private transport down

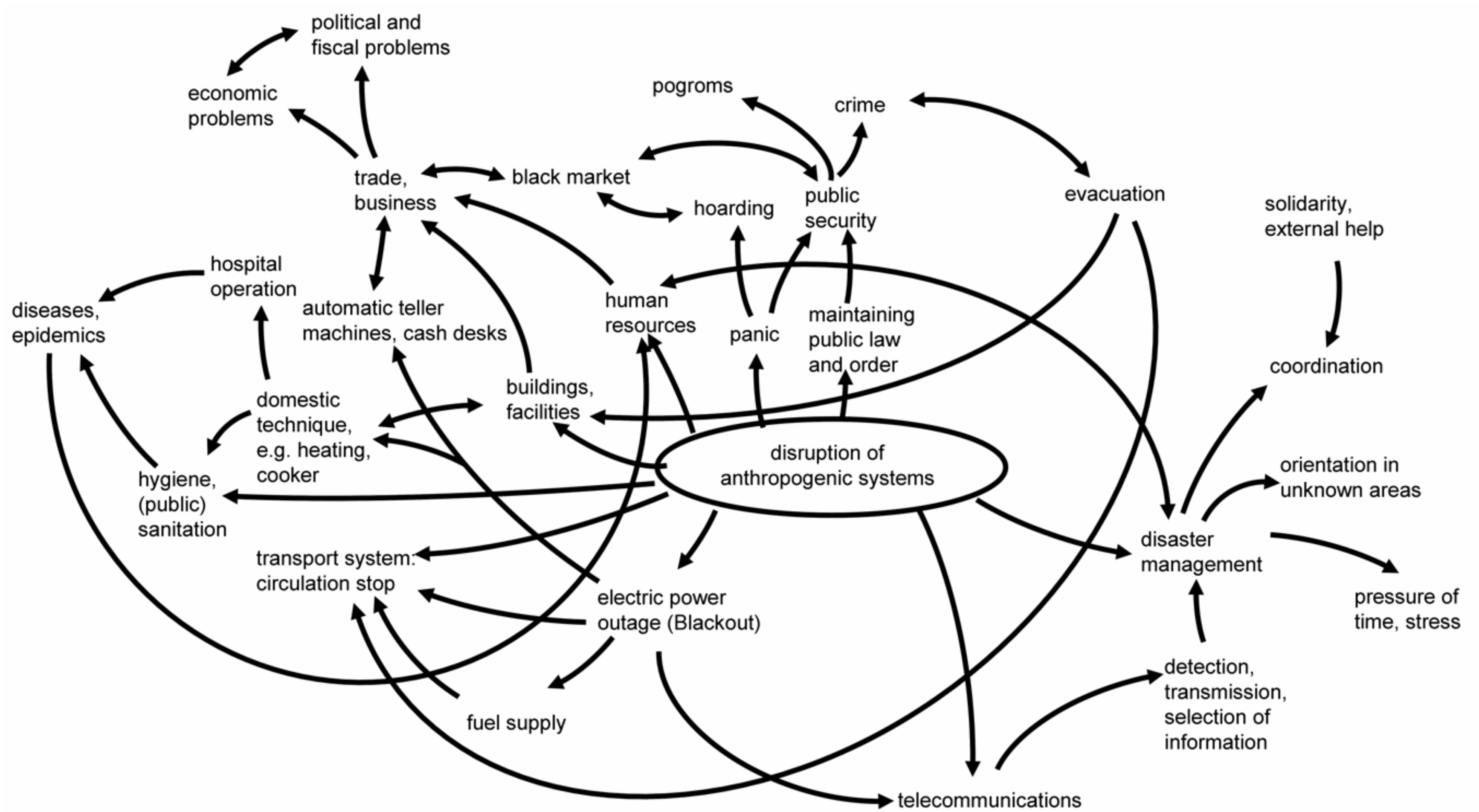
pumps without power

oil raffi-neries shut down

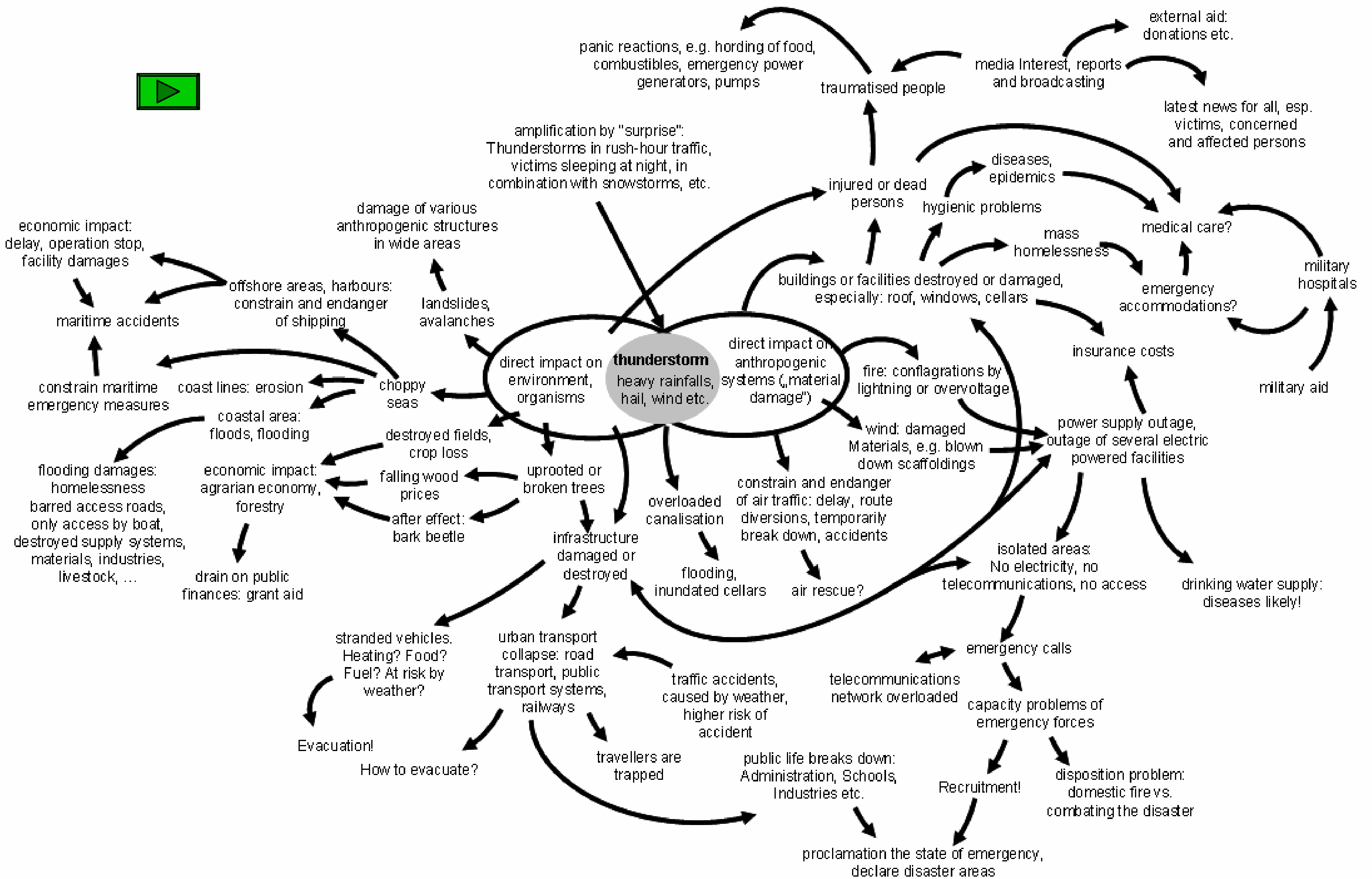
advisory to boil water

no electricity to boil water

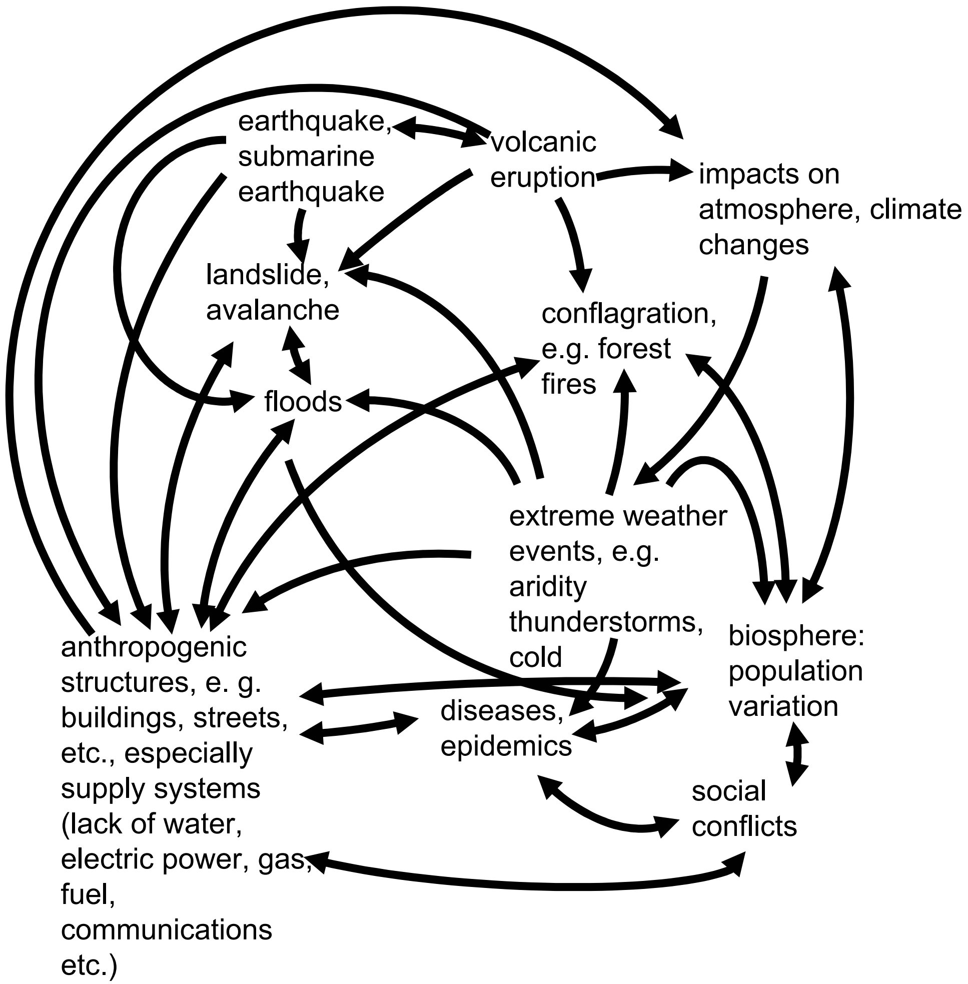
Common Elements of Disasters



Causality Network for Thunderstorms

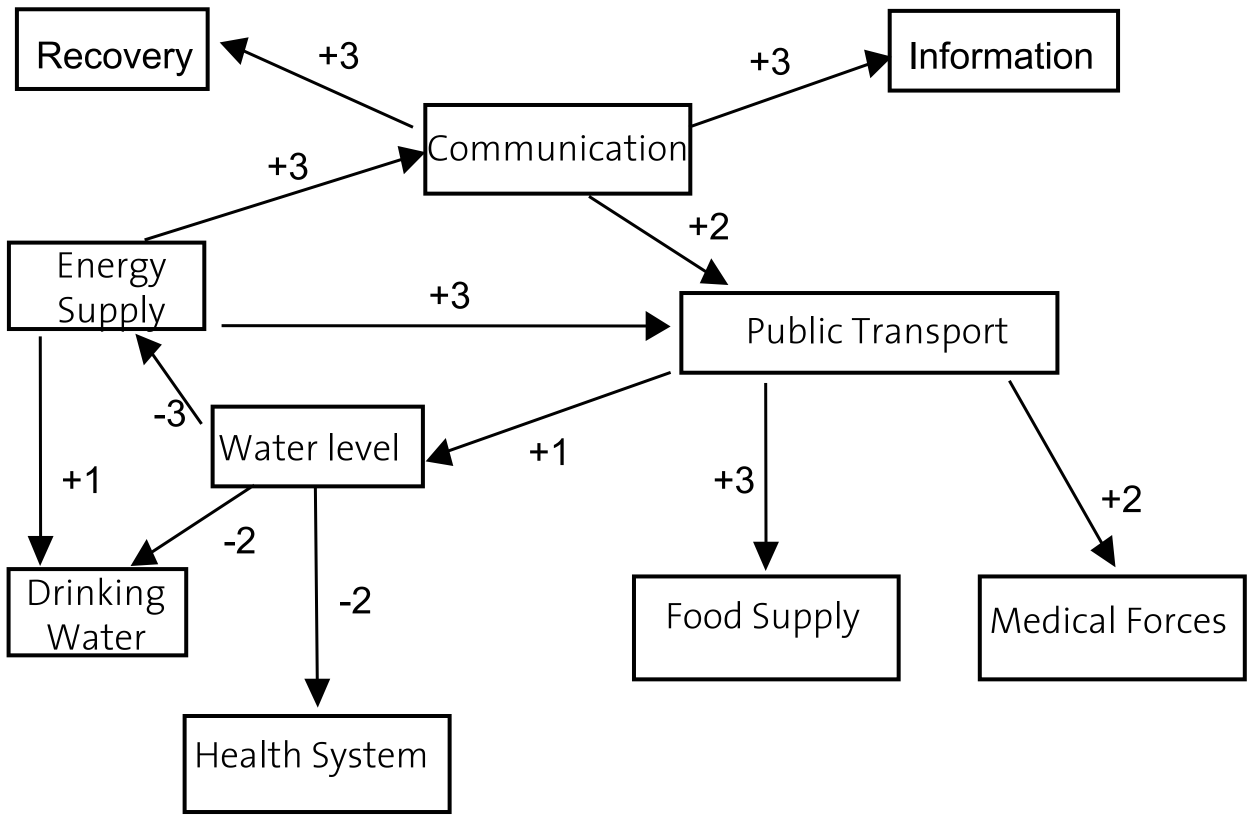


Disasters Cause Disasters

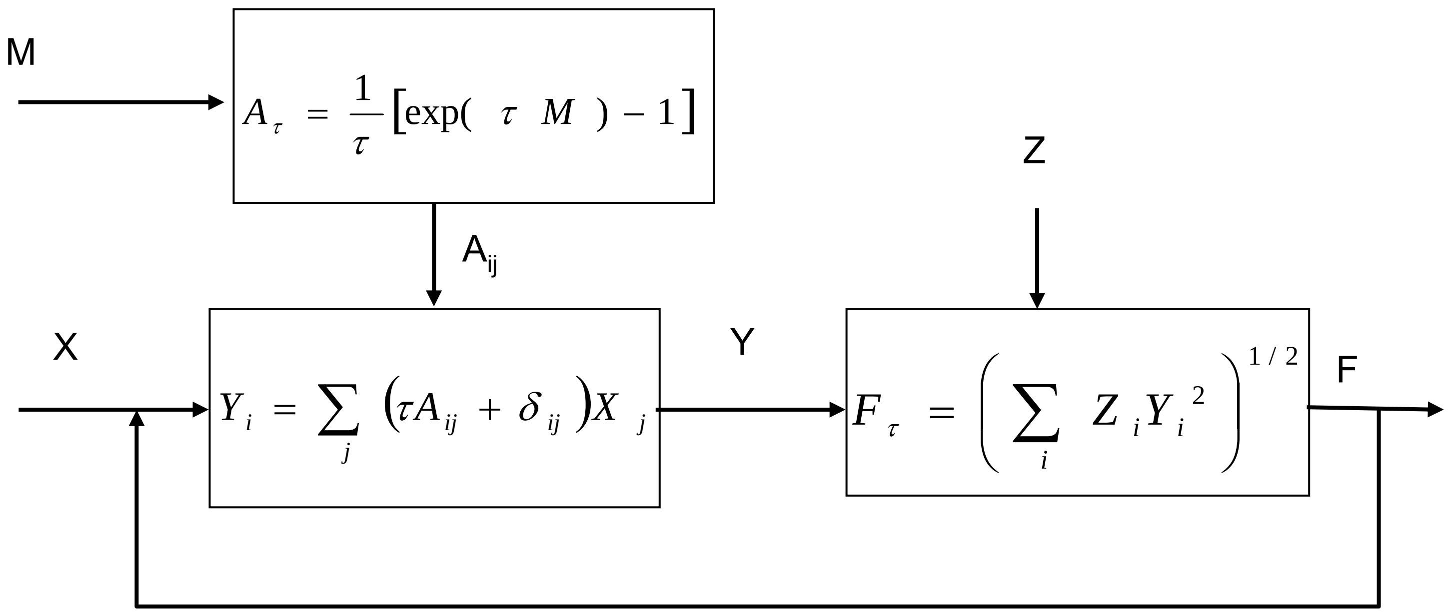


D. Helbing, H. Ammoser, and C. Kühnert: Disasters as extreme events and the importance of network interactions for disaster response management. Pages 319-348. in: S. Albeverio, V. Jentsch, and H. Kantz (eds.) *The Unimaginable and Unpredictable: Extreme Events in Nature and Society* (Springer, Berlin, 2005).

Causality Network of the Elbe Flooding 2002 (Detail)



Quantitative Analysis of Causality Networks



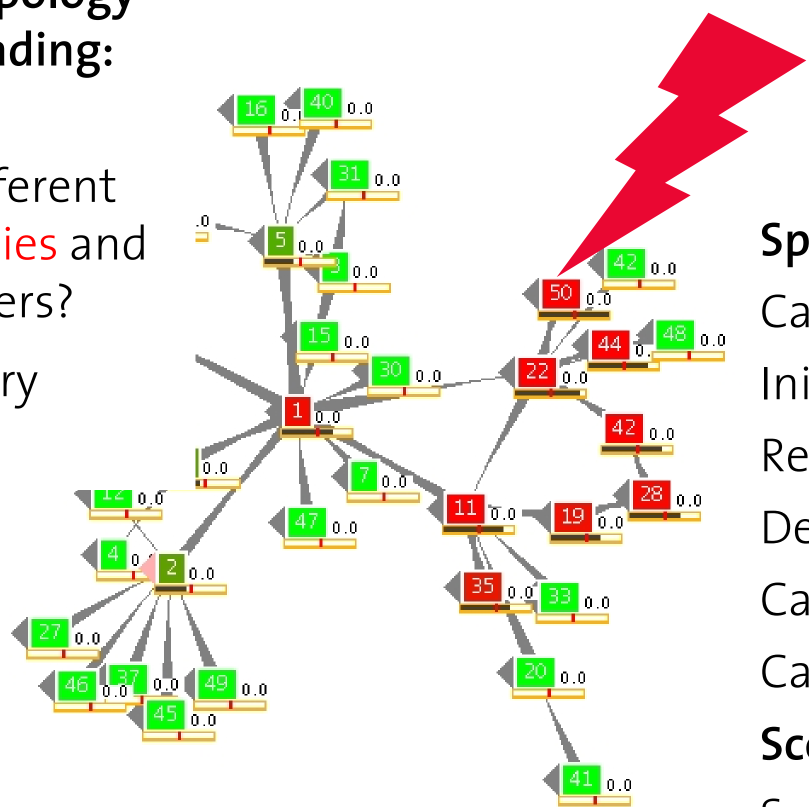
Identify the elements of the matrix M. Consider quantitative (data) and qualitative interactions {-3, ..., +3} and thus functional and structural characteristics of the causal networks for different means of disaster!

Modeling and Simulation of Disaster Spreading

Simulation of topology dependent spreading:

- What are the influences of different network topologies and system parameters?

- Optimal recovery strategies?



Buzna L., Peters K., Helbing D., Modelling the Dynamics of Disaster Spreading in Networks, Physica A, 2006

Spreading of disasters:

Causal dependencies (directed)

Initial event (internal, external)

Redistribution of loads

Delays in propagation

Capacities of nodes (robustness)

Cascade of failures

Scope of research:

Spreading conditions (network topologies, system parameters)

Optimal recovery strategies

Mathematical Model of Disaster Spreading

Node dynamics:

$$\frac{dx_i}{dt} = -\frac{x_i}{\tau} + \Theta\left(\sum_{j\neq i} \frac{M_{ij}x_j(t-t_{ij})}{f(O_i)} e^{-\beta t_{ij}/\tau}\right) + \xi_i(t)$$

x_i state of the node

$x_i = 0$ usual situation

$x_i > \theta_i$ node is destroyed

$$\Theta(x) = \frac{1-\exp(-\alpha x)}{1+\exp[-\alpha(x-\theta_i)]}$$

$$f(O_i) = \frac{aO_i}{1+bO_i}$$

θ_i node threshold

t_{ij} time delay

M_{ij} link strength

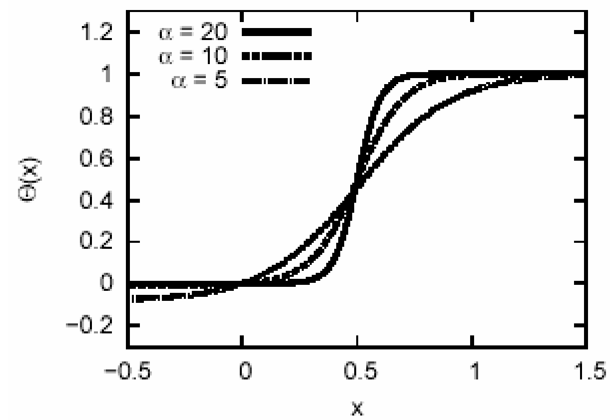
a, b, α, β fit parameters

$1/\tau$ healing rate

$\xi_i(t)$ internal noise

O_i node out-degree

Threshold function:



$$\Theta(x) = \frac{1-\exp(-\alpha x)}{1+\exp[-\alpha(x-\theta_i)]}$$

Node degree:

$$f(O_i) = \frac{aO_i}{1+bO_i}$$

We use a directed network, dynamical, bistable node models and delayed interactions along links.

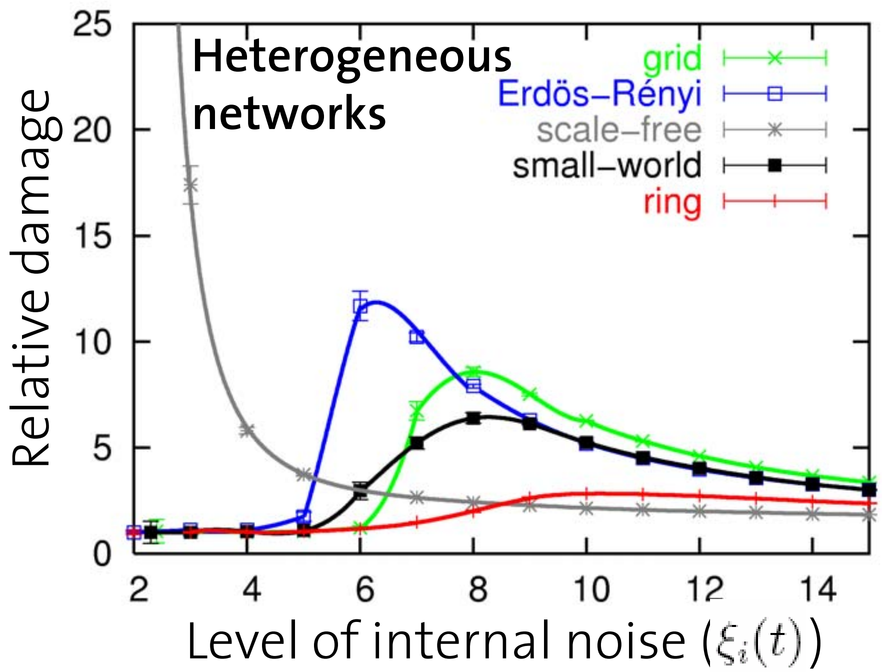
Failures Triggered by Internal Fluctuations

Coinciding, distributed, random failures:

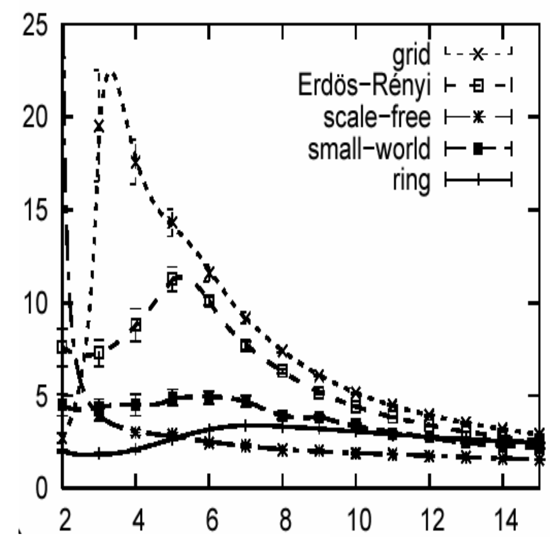
$$\frac{dx_i}{dt} = -\frac{x_i}{\tau} + \Theta\left(\sum_{j\neq i} \frac{M_{ij}x_j(t-t_{ij})}{f(O_i)} e^{-\beta t_{ij}/\tau}\right) + \xi_i(t)$$

L. Buzna, K. Peters, D. Helbing: Modeling the dynamics of disaster spreading in networks, *Physica A* **363**, 132-140 (2006)

Damage compared to an "unconnected network":



Homogeneous networks



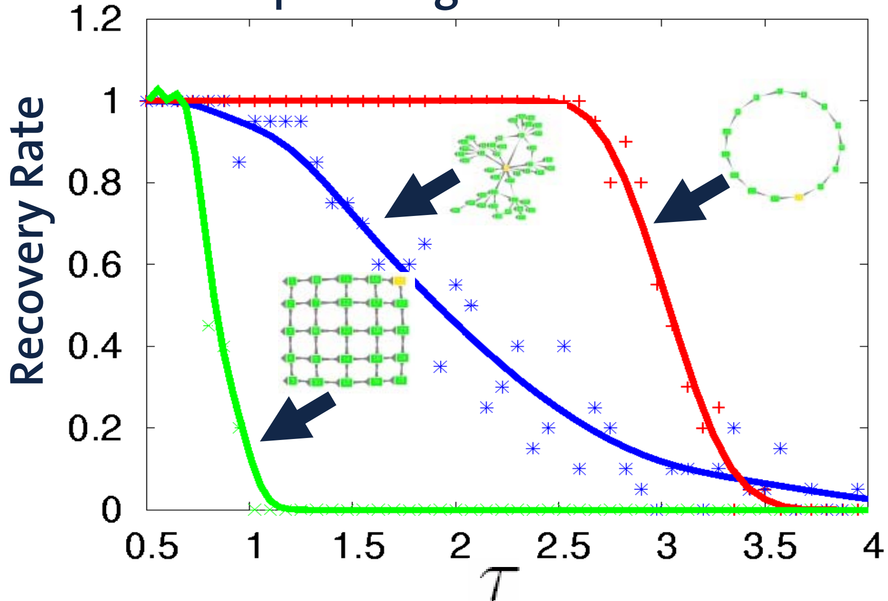
Connectivity is an important factor (in a certain region).

Phase Transition in Disaster Spreading

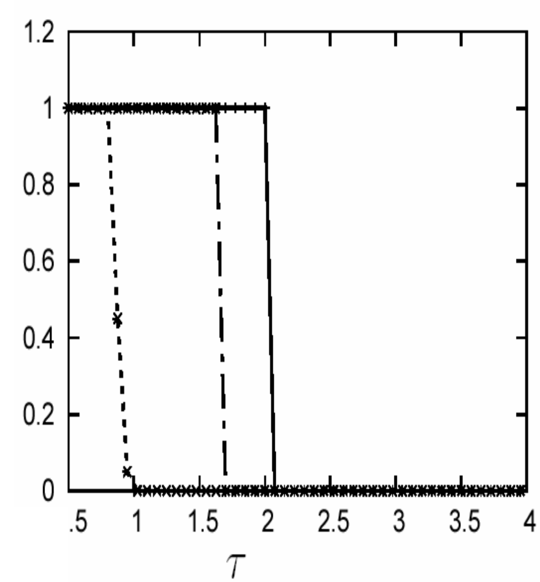
Node robustness vs. failure propagation:

$$\frac{dx_i}{dt} = -\frac{x_i}{\tau} + \Theta\left(\sum_{j\neq i} \frac{M_{ij}x_j(t-t_{ij})}{f(O_i)} \mathrm{e}^{-\beta t_{ij}/\tau}\right) + \xi_i(t)$$

Spreading Threshold



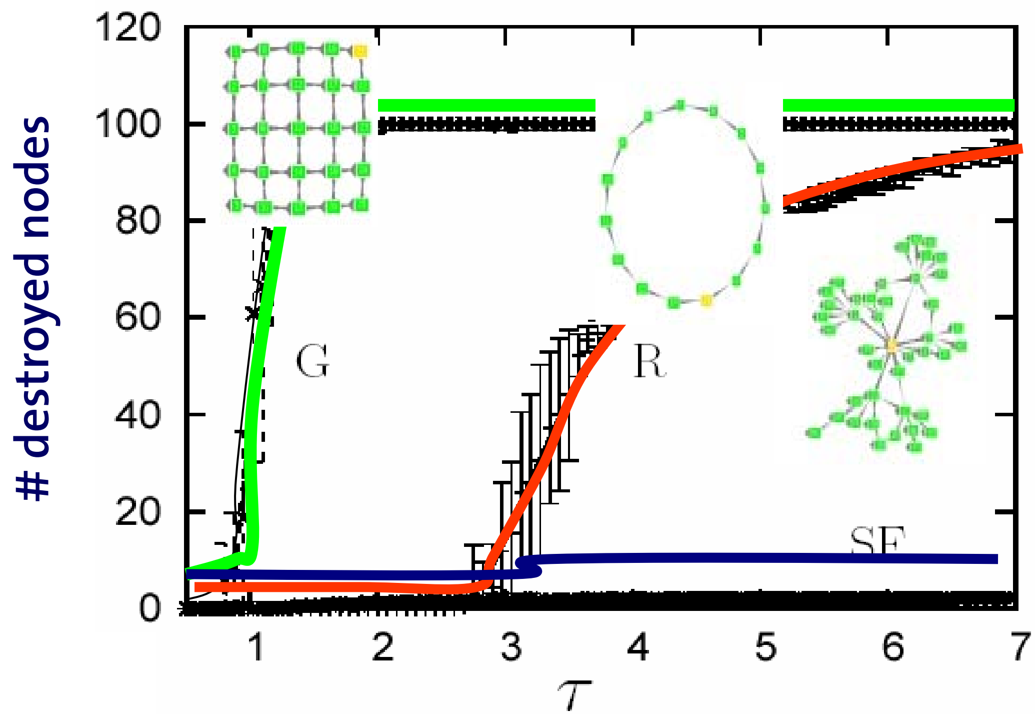
Homogeneous networks



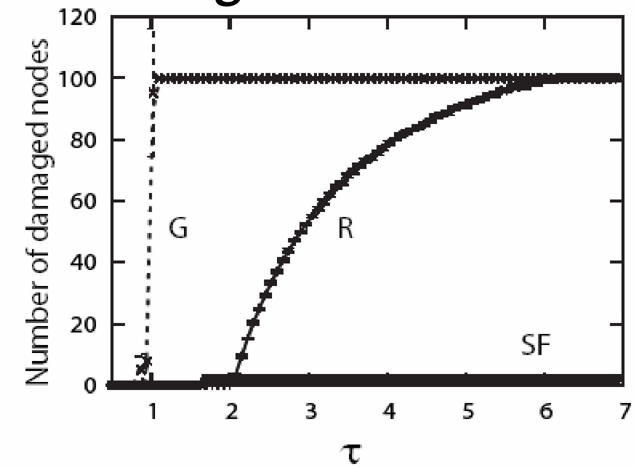
➡ **We found a critical threshold for the spreading of disasters in networks.**
Topology and parameters are crucial.

Topology and Spreading Dynamics

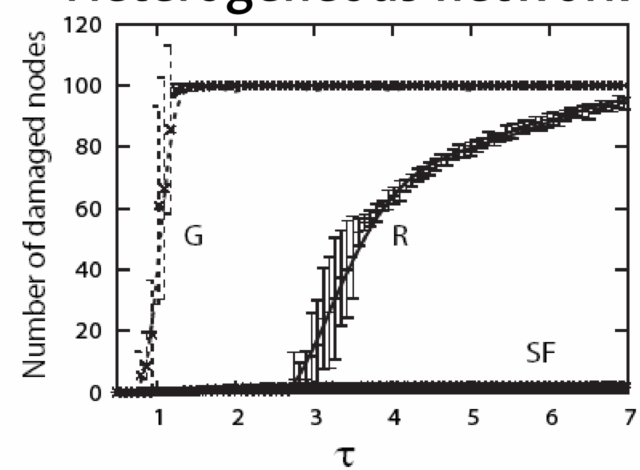
Example: 100 nodes, average state after t=300



Homogeneous network



Heterogeneous network



We found a topology dependent „velocity“ o
failure propagation.
Spreading in scale-free networks is slow.

K. Peters, L. Buzna, D. Helbing: Modelling of cascading effects and efficient response to disaster spreading in complex networks, International Journal of Critical Infrastructures, in print (2007).

Modelling the Recovery of Networks

1. Mobilization of external resources:

$$r(t) = a_1 t^{b_1} e^{-c_1 t}$$

2. Formulation of recovery strategies as a function of

- the network topology

- the level of damage

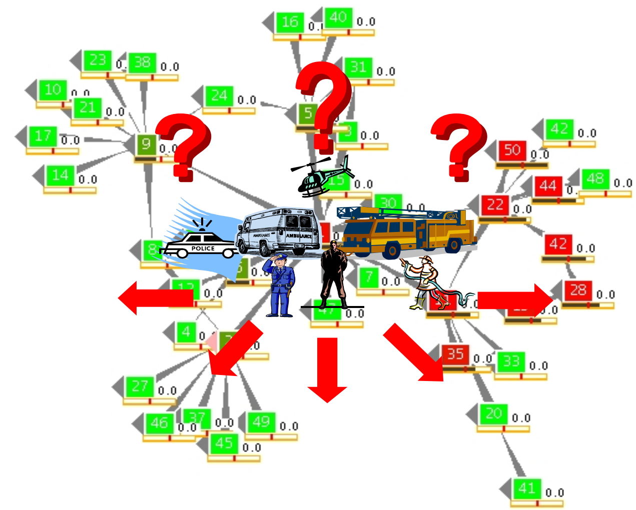
$$\frac{1}{\tau_i(t)} = \frac{1}{(\tau_{start} - \beta_2) e^{-\alpha_2 R_i(t)} + \beta_2}$$

3. Application of resources in nodes

Parameters:

t_D time delay in response

R disposition of resources



$R_i(t)$ cumulative number of resources deployed at node *i*

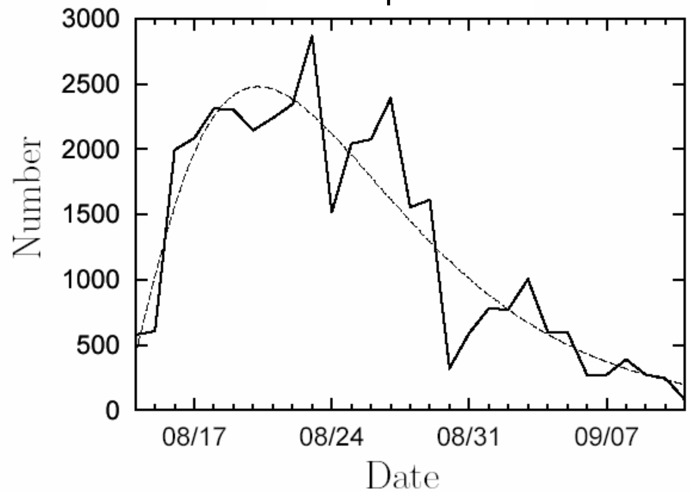
τ_{start} initial intensity of recovery process

α_2 β_2 fit parameters

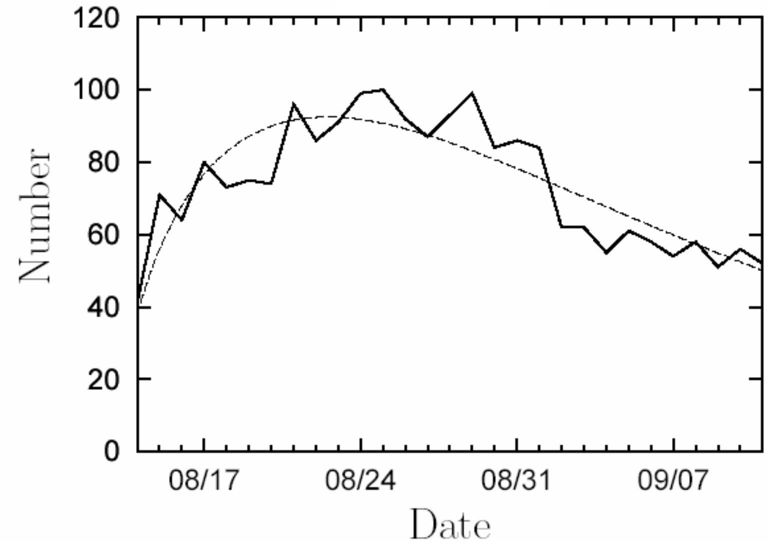
Mobilization of Resources

Example: Mobilization during the Elbe flood 2002:

Manpower



Vehicles



Mobilization of resources (time dependent)

External resources become available after a certain response time delay T_D

During mobilization the number of resources increases

Later a phase of demobilization occurs

Number of available resources *r(t)*:

$$\boldsymbol{R}(t) = a_1 t^{b_1} e^{-c_1 t}$$

a_1, b_1, c_1 are fit parameters

Recovery Strategies

Application of external resources in nodes:

$$\tau(t) = (\tau_{start} - \beta) exp^{-\alpha R_i(t)}$$

$R_i(t)$ cumulative number of resources deployed at node *i*

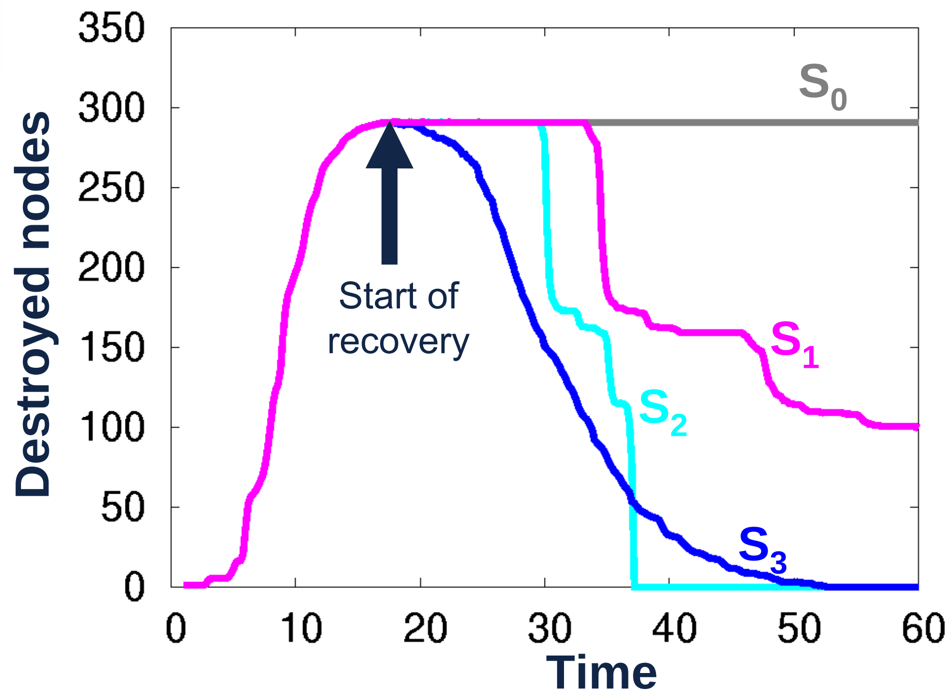
τ_{start} time to start healing

α β fit parameters

Formulation of recovery strategies as a function of the

- network topology
- level of damage

Application of resources in a scale-free network



S_0 – no recovery

S_1 – uniform deployment

S_2 – priority1: destroyed nodes
priority2: damaged nodes

S_3 – out-degree based deployment

How to Distribute Available Resources ?

Formulation of recovery strategies, based on information :

S_0 no recovery

Topology information only:

S_1 uniform deployment

S_2 out degree based dissemination

Damage information:

S_3 uniform reinforcement of challenged nodes $(x_i > 0)$

S_4 uniform reinforcement of destroyed nodes $(x_i > \theta_i)$

Damage & topology information:

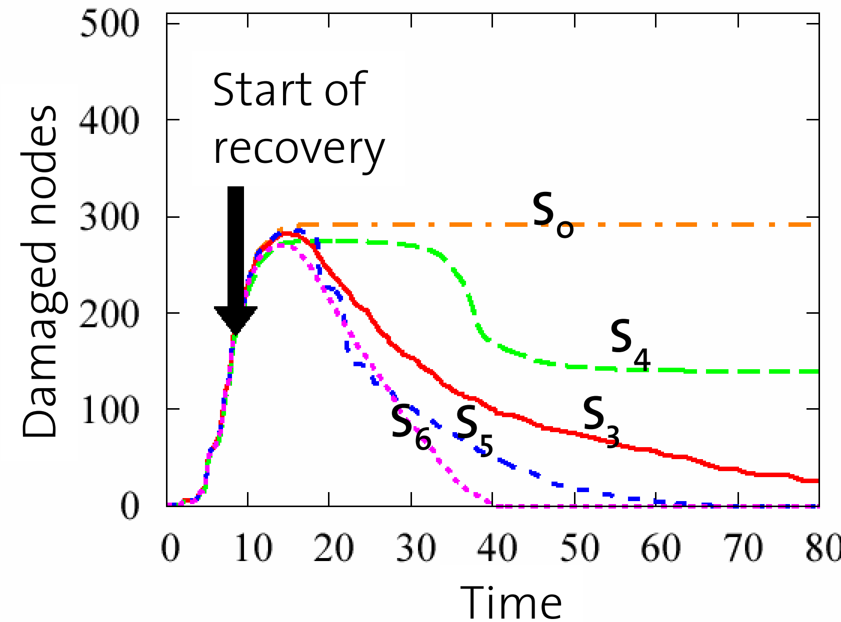
S_5 targeted reinforcement of highly connected nodes

1st priority: fraction q to hub nodes

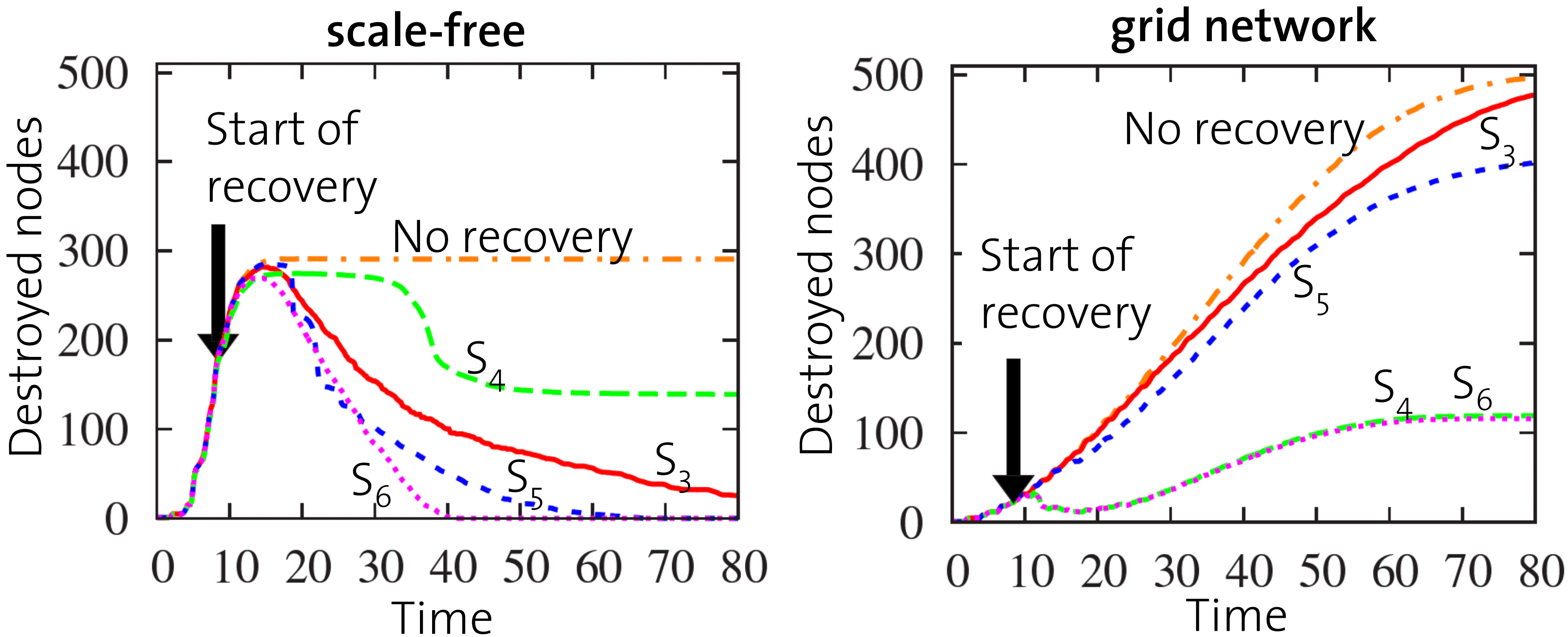
2nd priority: fraction 1-q according to S_4

S_6 out-degree based targeted reinforcment of destroyed nodes

Application of resources to a scale-free network



Recovery of Networks



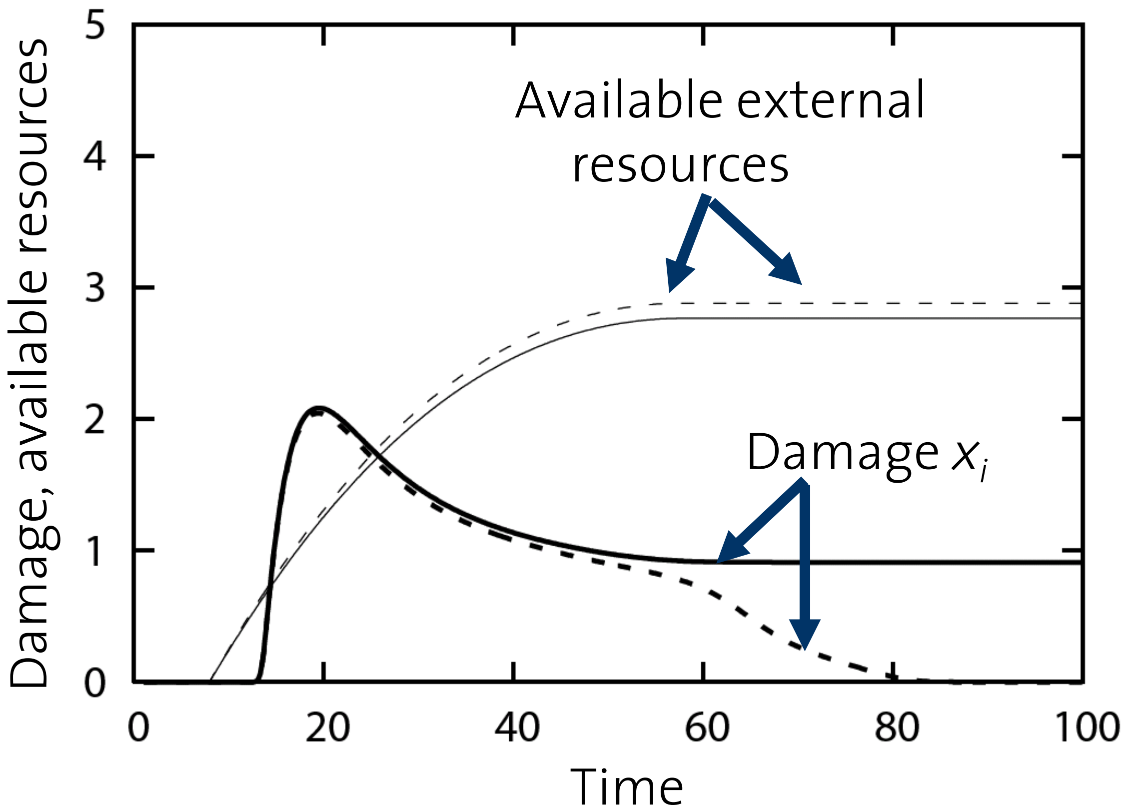
Parameters: Network topology

time delay in response $t_D = 8$

disposition of resources $R = 1000$

L. Buzna, K. Peters, H. Ammoser, Ch. Kuehnert and D. Helbing: Efficient response to cascading disaster spreading, *Physical Review E* **75**, 056107 (2007)

Behaviour of a Node for Sufficient and Insufficient Resources



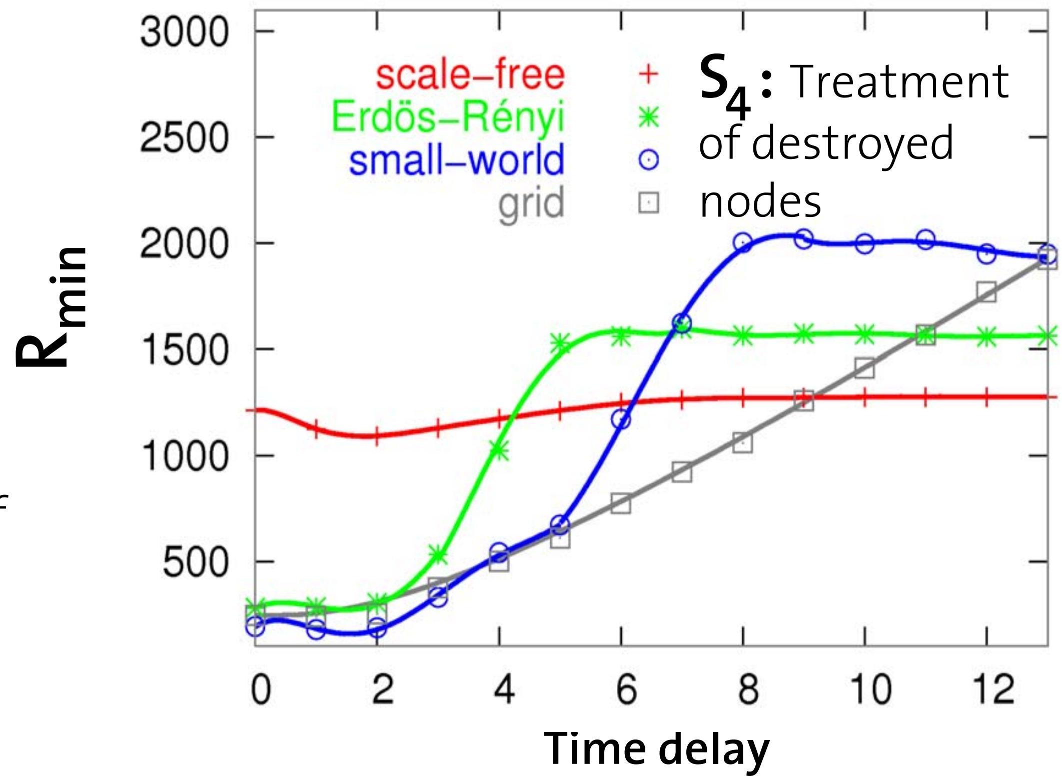
K. Peters, L. Buzna, and D. Helbing (2007) Modelling of cascading effects and efficient response to disaster spreading in complex networks (in print)

Minimum Quantity of Resources R_{min} for Recovery

Given: Amount of resources, mobilized with certain delay.

Is the network able to recover?

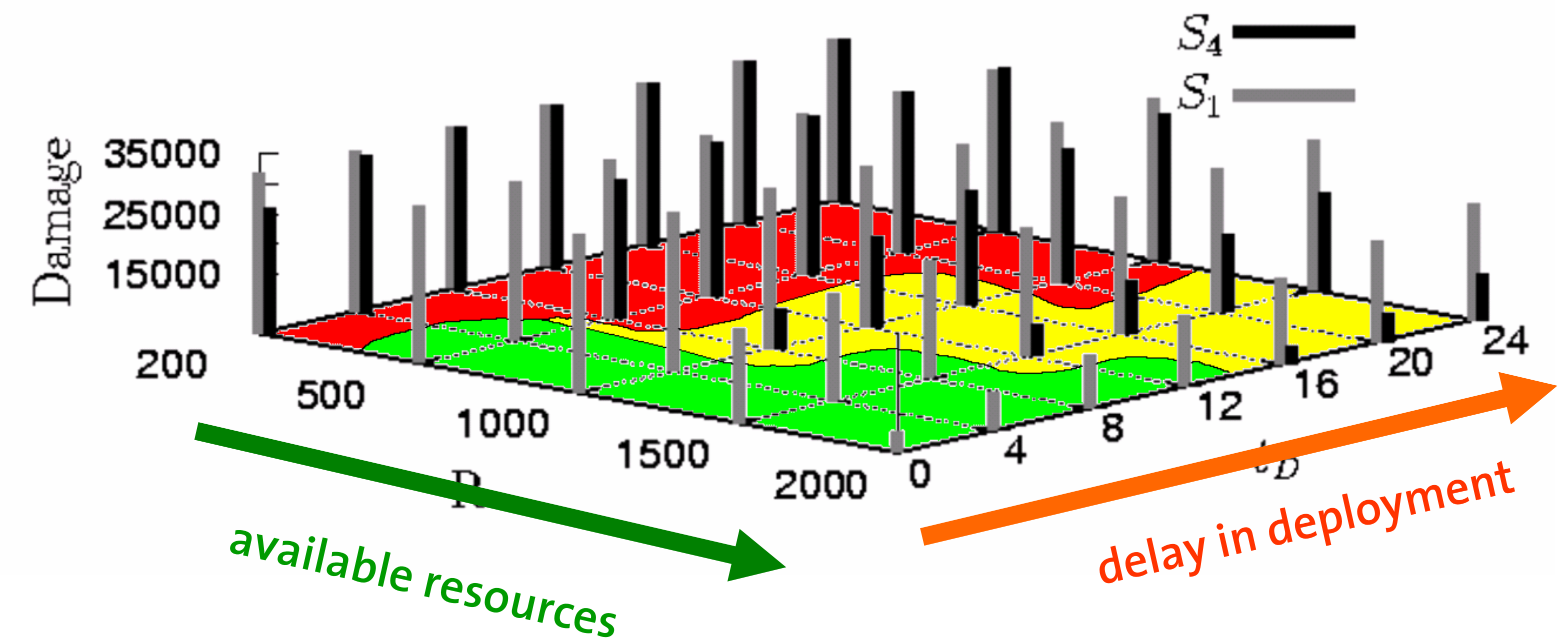
Minimum quantity of resources needed to recover a challenged network as a function of the response time delay and network topology



Recovery (in reasonable time) is not always possible.

Recovery of Networks: When Does Strategy Matter?

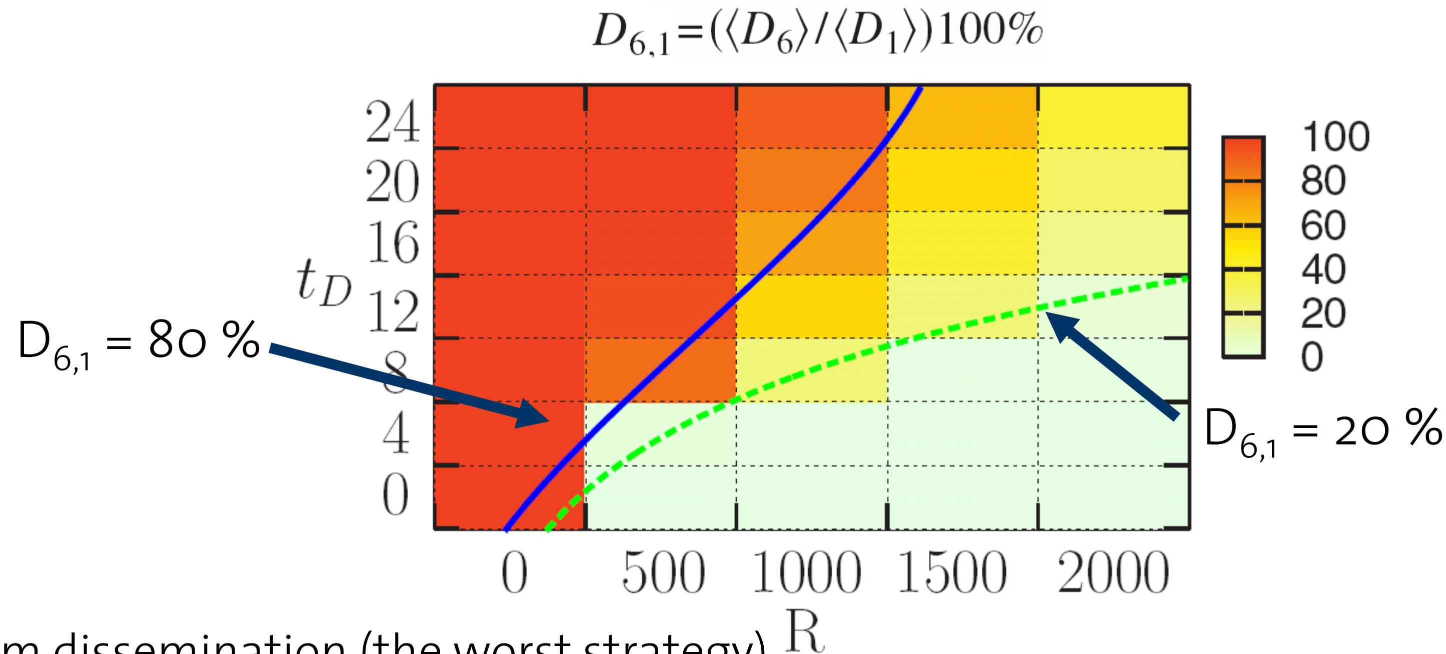
Comparison of efficient and inefficient strategies:



- The delay of recovery activities is crucial.
- Optimization of recovery strategies is promising in certain parameter regions.

Comparison of Efficient and Inefficient Strategies

Relative difference in damage between S_6 and S_1

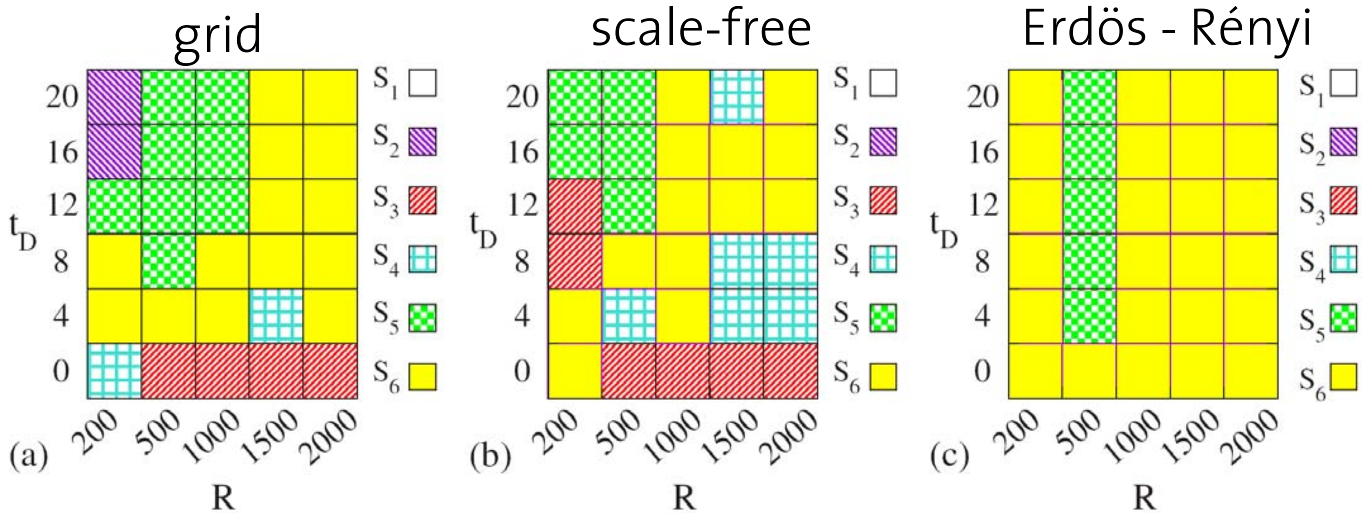


S_1 - uniform dissemination (the worst strategy)

S_6 – out – degree based targeted reinforcement of destroyed nodes (the best strategy)

1. The promptness of recovery activities has a crucial influence on their efficiency
2. Optimization of protection strategies is possible in certain parameter regions

Most Efficient Strategies



There is no unique optimal response strategy:

1. Strategies based on the network structure has been proved as a most suitable for scale-free structures.
2. Strategies based on the damage information are more appropriate for regular networks.
3. The situation in Erdős-Rényi and small-world networks depends on t_D (short t_D => damage based strategies)

   (large t_D => network structure based strategies)

Mixed Recovery Strategies

Objectives:

- Minimal average damage
- Minimal sufficient quantity of resources

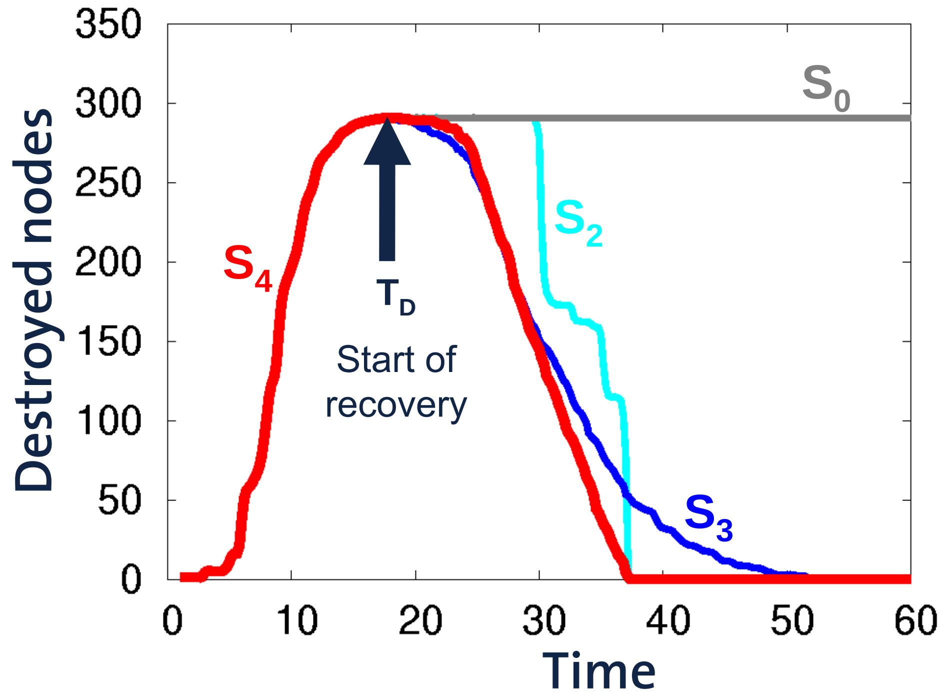
Parameters:

- **R** overall disposition of resources
- t_D time delay of recovery
- Network topology

Methods:

- Mixing of basic strategies
- Switching between strategies in time

Application of resources (R = 2000) on scale-free network



S_4 : 57 % S_2, 43 % S_3

Network-Dependence of Best Strategy

Strategies based on the network structure have been proven most suitable for scale-free structures.

Strategies based on information about the degree of damage are more appropriate for regular networks.

The situation in Erdös-Rényi and small-world networks depends on the response time t_D

(short t_D => orient at damage)

(large t_D => orient at network structure)

Critical Infrastructures and Their Vulnerability

- "Critical Infrastructures consist of those physical and information technology facilities, network services and assets which, if disrupted or destroyed, would have a serious impact on the health, safety, security or economic well-being of citizens or the effective functioning of governments".

  (Commission of the European Communities in 2004)

- A system is said to be vulnerable if its functioning can be significantly reduced by intentional or non-intentional means.

Level of Vulnerability

System's functioning

$$\mathrm{L} = \frac{df}{du}$$

System's failure

Generation of Traffic in the Computer

In the case of freeways (no choice of different travel modes/ means of transport), the classical 4-step model reduces to the following 3 steps:

1. Trip generation (overall traffic volume generated per hour)
2. Trip distribution (OD choice with multinomial logit model, exponentially distributed as function of travel time)
3. Traffic assignment (based on travel time, distribution over alternative routes according to the Wardrop principle)

Travel time on link ***l*** is modeled by the classical capacity constraint function

$$T_l(q_l) = T_l^0[a(1+(q_l/k_l)^b]$$

Topological Analysis

Efficiency

$$E\,[G] = \frac{1}{N(N-1)} \sum_{i,j \in G} \frac{1}{d_{ij}}$$

dij – shortest path between nodes i and j

Edge Information Centrality

$$IC_{ij} = \frac{\Delta E}{E} = \frac{E[G] - E[G']}{E[G]}$$

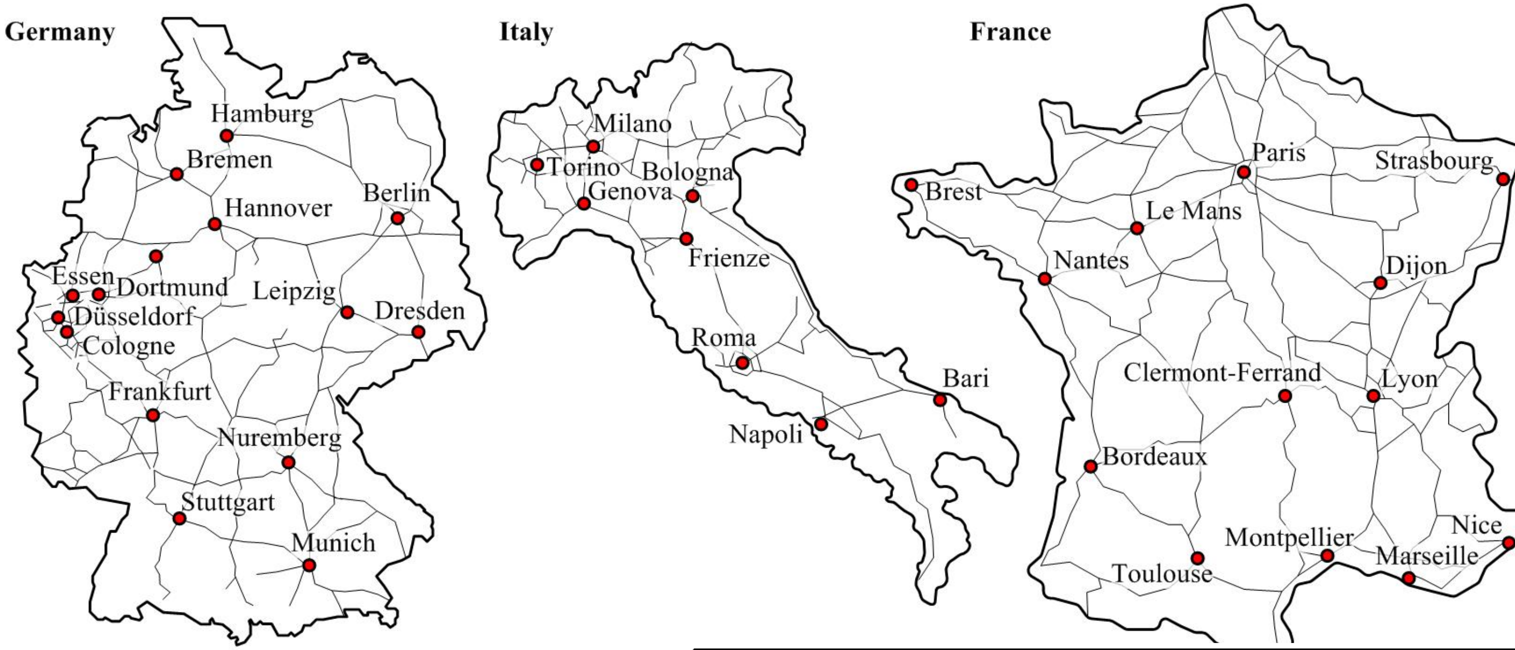
E(G) and E(G’) is the efficiency before and after the links’ removal, respectively

Edge Betweenness Centrality

$$b_{ij} = \frac{n_{ij}}{(N-1)(N-2)}$$

n_{ij} number of shortest pathes between city nodes which pass through the edge connecting nodes i and j

Case Study: The Italian German, and French Highways

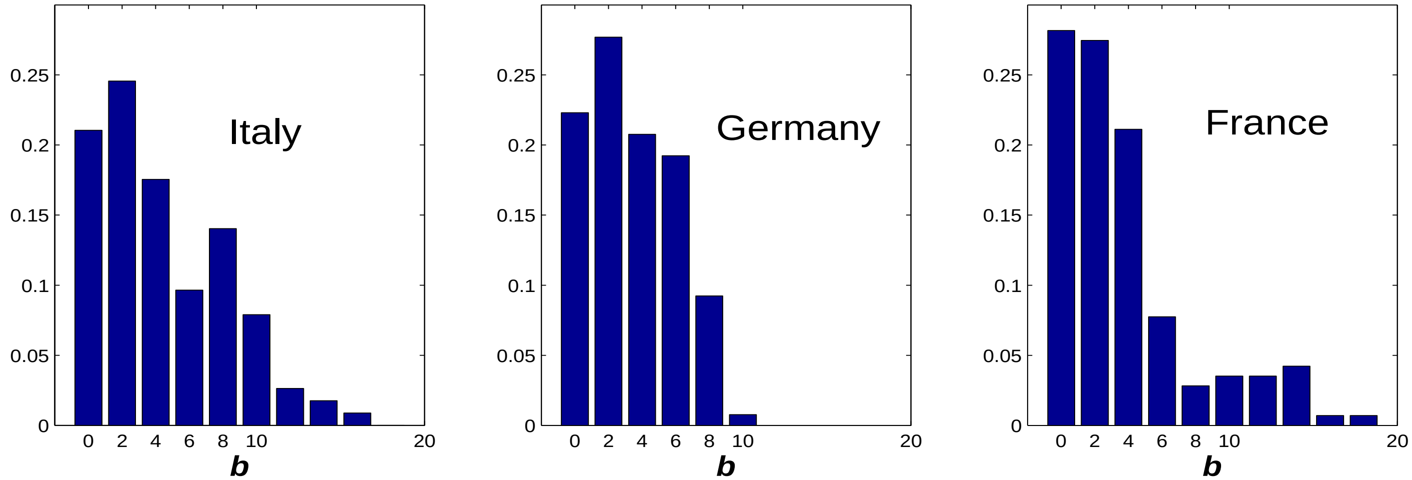


For each country we chose a subset of the network that includes the highways connecting 29 of the most populated cities

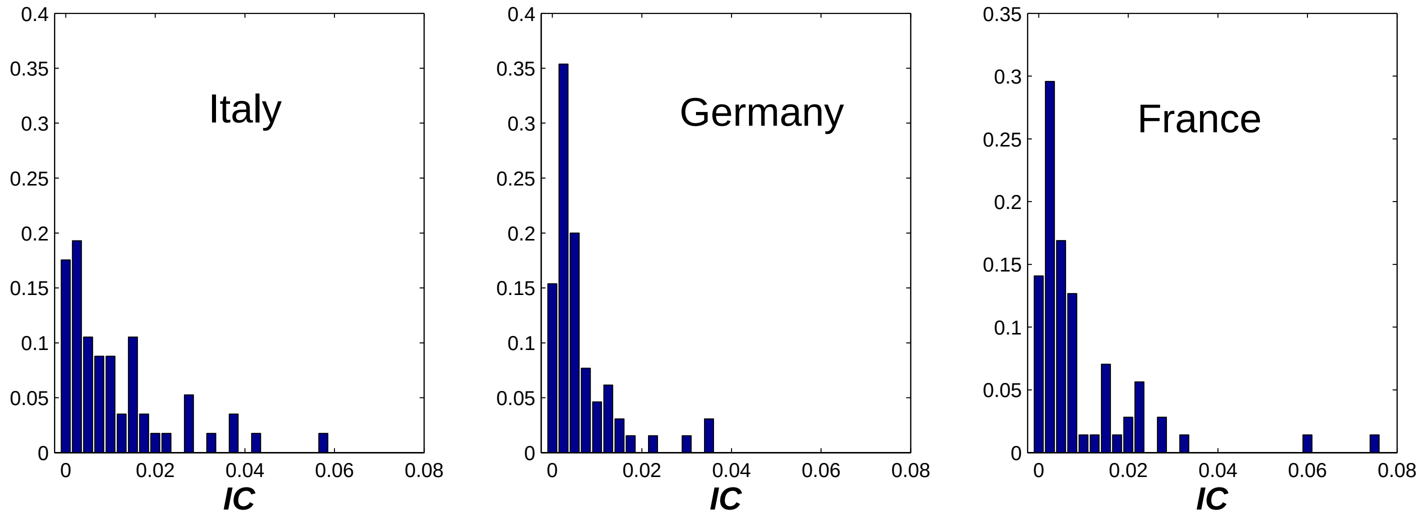
	Italy	Germany	France
population	10701491	17366502	8381434
nodes	43	43	52
road's number	57	65	71
total road's length (km)	4946	5453	8194
average road's length (km)	86	83	115
$<$ degree $>$	2.6	3.0	2.7
$< b_{ij} >$	0.0451	0.0328	0.0332

Topological Analysis

Edge Betweenness Centrality



Edge Information Centrality



Functional Vulnerability of a Freeway System

Topological efficiency

$$E[G] = \frac{1}{N(N-1)} \sum_{i,j \in G} \frac{1}{d_{ij}}$$

dij – shortest path between nodes *i* and *j*

$$\mathrm{L}_T^{(u)} = \frac{\Delta E}{E} = \frac{E[G(0)] - E[G(u)]}{E[G(0)]}$$

Flow related analysis

Efficiency

$$E_F = \frac{1}{N(N-1)} \sum_{ij \in OD} \frac{1}{C_{ij}}$$

Cost Function

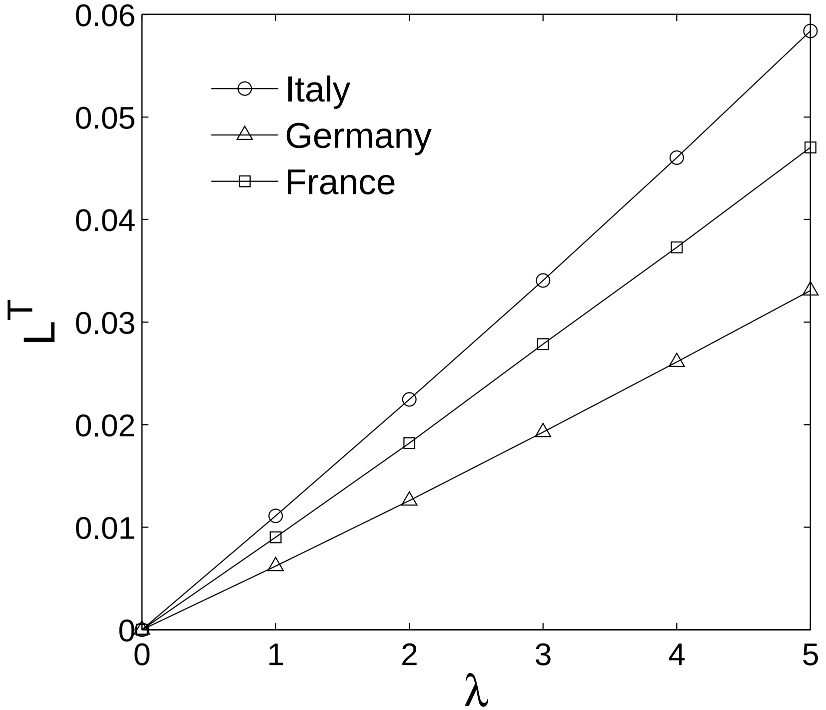
$$C_{ij} = q_{ij} T_{ij}$$

Quality of service

$$QoS = \frac{E_F}{E_{F_{max}}} = \frac{\sum_{ij} \mathrm{C}_{ij}{}^{-1}}{\sum_{ij} (\mathrm{C}_{ij}^{\min})^{-1}}$$

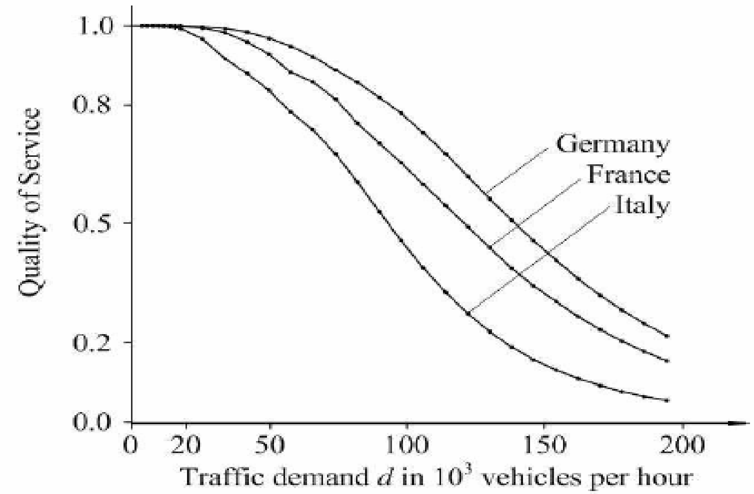
$$L_F^{(u)} = 1 - \left\langle \frac{\mathrm{QoS}^{(u)}}{\mathrm{QoS}(0)} \right\rangle$$

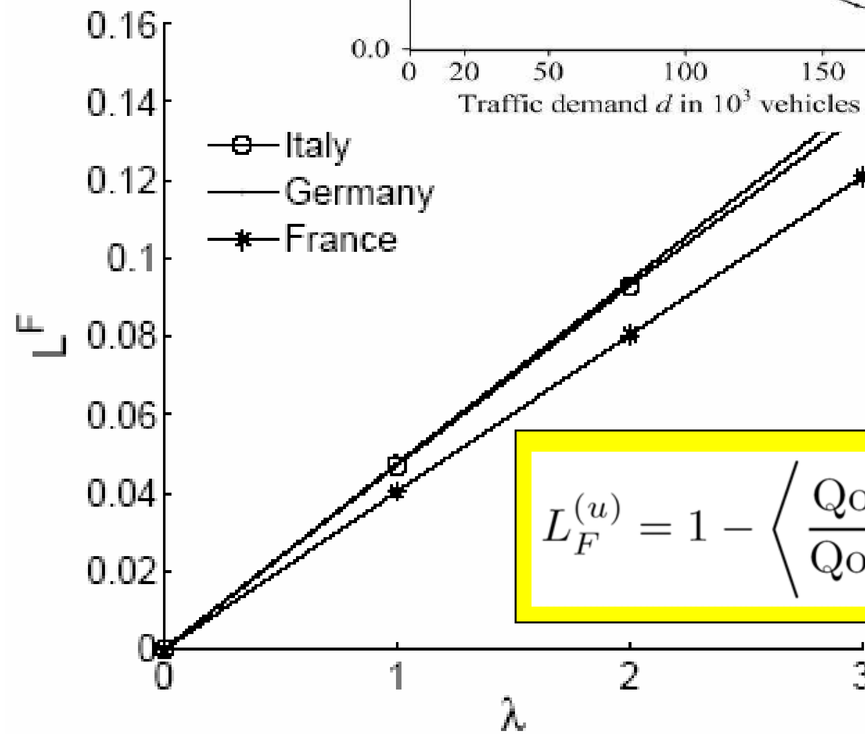
Topological Vulnerability and Flow-Related Vulnerability



$$L_T^{(u)} = \frac{\Delta E}{E} = \frac{E[G(0)] - E[G(u)]}{E[G(0)]}$$

λ is the number of links simultaneously removed from the network





$$L_F^{(u)} = 1 - \left\langle \frac{\mathrm{QoS}^{(u)}}{\mathrm{QoS}(0)} \right\rangle$$

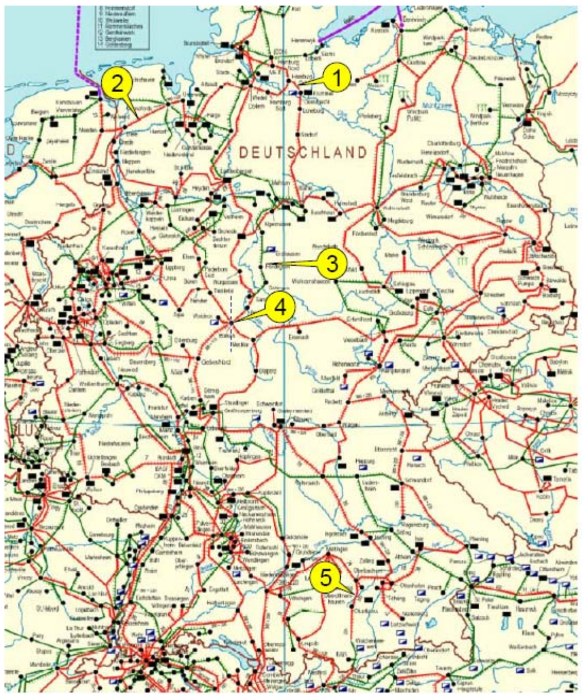
Blackouts and Cascading Effects in Electricity Networks

New York, August 14, 2003

Rome, September 28, 2003

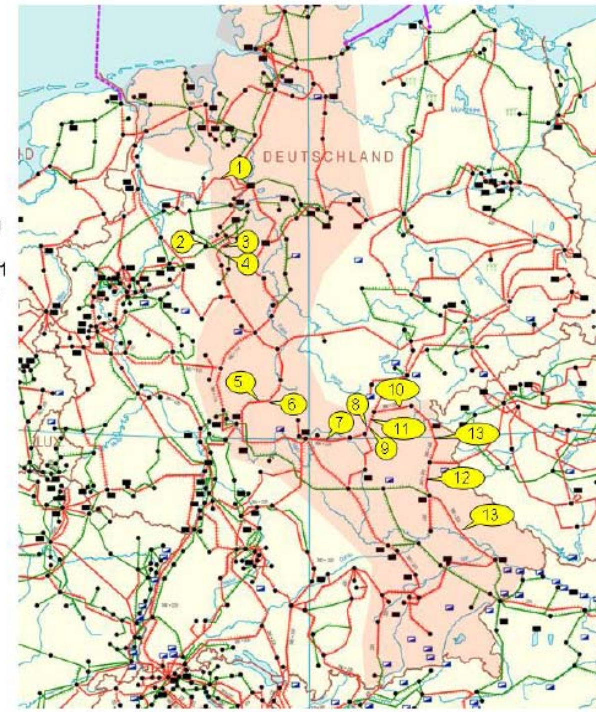
Blackouts and Cascading Effects in Electricity Networks

State of the power grid shortly before the incident



Sequence of events on November 4, 2006

Nr.	Zeit	kV	Leitung
1	22:10:13	380	Wehrendorf-Landesbergen
2	22:10:15	220	Bielefeld/Ost-Spexard
3	22:10:19	380	Bechterdissen-Elsen
4	22:10:22	220	Paderborn/Süd-Bechterdissen/Gütersloh
5	22:10:22	380	Dipperz-Großkrotzenburg 1
6	22:10:25	380	Großkrotzenburg-Dipperz 2
7	22:10:27	380	Oberhaid-Grafenrheinfeld
8	22:10:27	380	Redwitz-Raitersaich
9	22:10:27	380	Redwitz-Oberhaid
10	22:10:27	380	Redwitz-Etzenricht
11	22:10:27	220	Würgau-Redwitz
12	22:10:27	380	Etzenricht-Schwandorf
13	22:10:27	220	Mechlenreuth-Schwandorf
14	22:10:27	380	Schwandorf-Pleinting



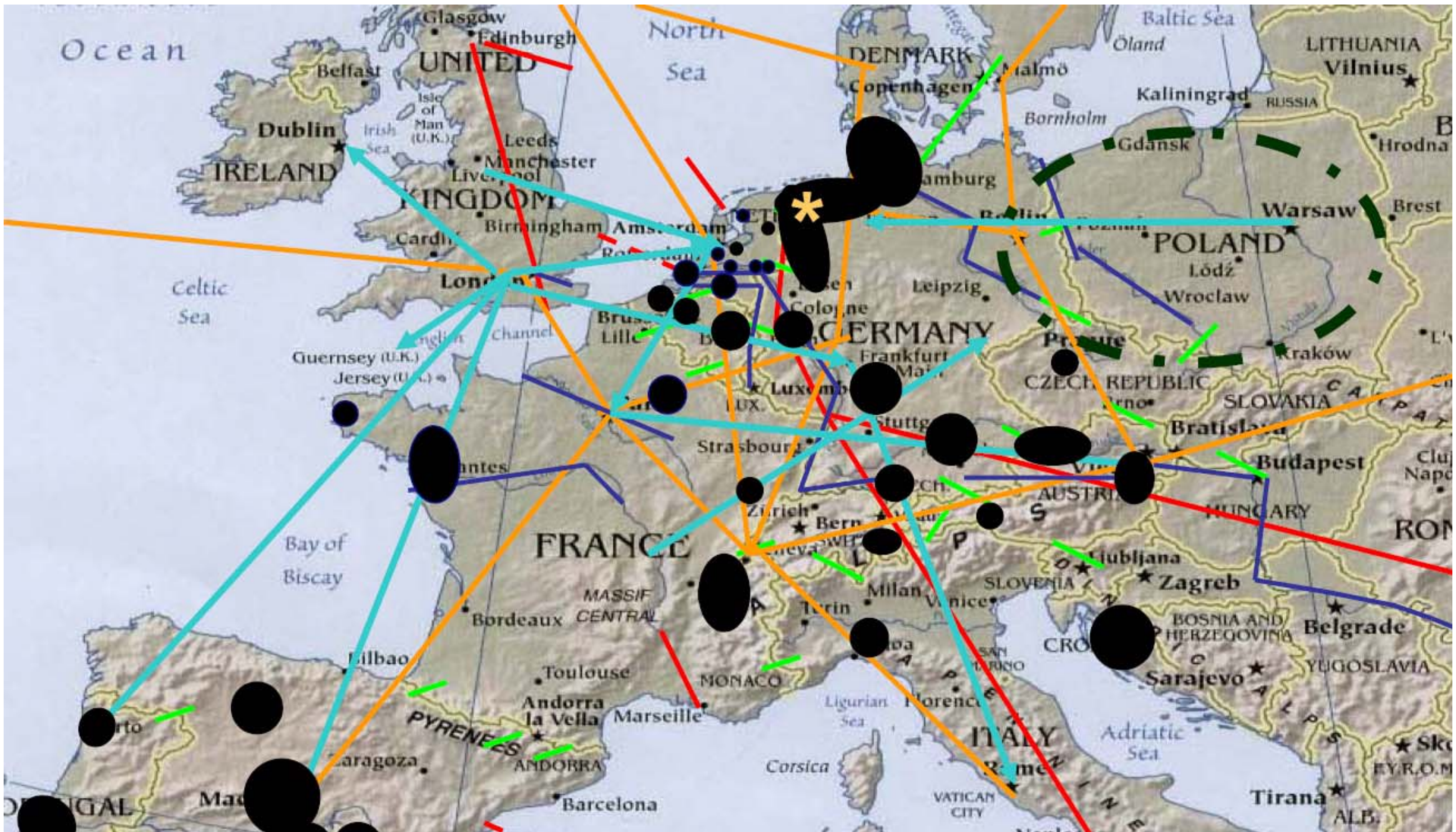
1,3,4,5 – lines switched off for construction work

2 – line switched off for the transfer of a ship by Meyer -Werft

E.ON Netz's report on the system incident of November 4, 2006, E.ON Netz GmbH

Blackouts and Cascading Effects in Electricity Networks

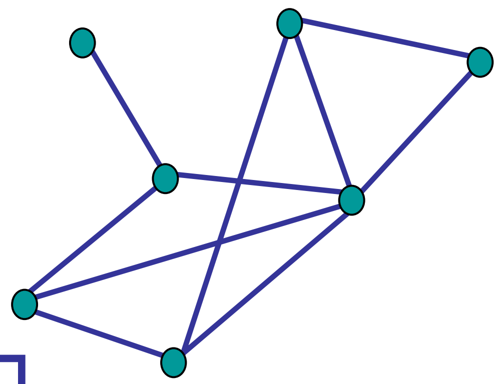
Failure in the continental European electricity grid on November 4, 2006



EU project IRRIIS: E. Liuf (2007) Critical Infrastructure protection, R&D view

Dynamic Model of Cascading Failures

Network:



$\mathcal{N}$ set of nodes

$\mathcal{L}$ set of links

$\boldsymbol{W}$ adjacency matrix ($W_{ij} \geq 0$, link weight)

Model dynamics:

$$n_i(t+1) = \sum_{j=1}^{\mathcal{N}} T_{ij} n_j(t) + n_i^{\pm}$$

(Master equation)

$n_i(t)$ number of particles hosted by node *i* at the time *t*

$T_{ij} = W_{ij}/w_j,\ w_j = \sum_{\ell=1}^{\mathcal{N}} W_{\ell j}$

$n_i^{\pm} > 0$ node is source, $n_i^{\pm} < 0$ node is sink

I. Simonsen, L. Buzna, K. Peters, S. Bornholdt, D. Helbing, Stationary network load models underestimate vulnerability to cascading failures, 2007, submitted, eprint : http://arxiv.org/pdf/0704.1952

Stationary and Dynamic Models of Cascading Failures

Model normalization:

$\rho_i(t) = n_i(t)/N$ nodal particle density

$c_i(t) = \rho_i(t)/w_i$ utilization of outflow capacity

$j_i^{\pm} = n_i^{\pm}/(N w_i)$ sinks and sources term

Dynamic model: →

$$\boldsymbol{c}(t+1) = \boldsymbol{\mathcal{T}}\boldsymbol{c}(t) + \boldsymbol{j}^{\pm}$$

$c_i^{(0)}(\infty) = 1/(N w_i)$ stationary solution for $\boldsymbol{j}^{\pm} = 0$, otherwise

Stationary model: →

$$\boldsymbol{c}(\infty) = \boldsymbol{c}^{(0)}(\infty) + (\mathbf{1} - \boldsymbol{\mathcal{T}})^{+}\boldsymbol{j}^{\pm}$$

$(\mathbf{1} - \boldsymbol{\mathcal{T}})^{+}$ generalized inverse of matrix $\mathbf{1} - \boldsymbol{\mathcal{T}}$

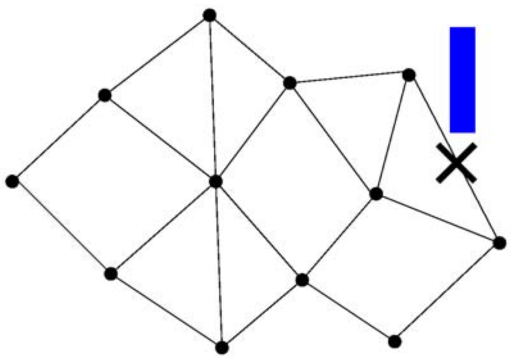
$C_{ij}(t) = W_{ij} c_j(t)$ current on link from *i* to *j*

Link flow:

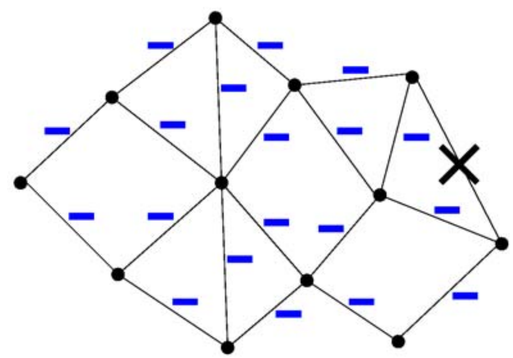
$L_{ij}(t) = C_{ij}(t) + C_{ji}(t)$

Stationary and Dynamic Models for Cascading Failures

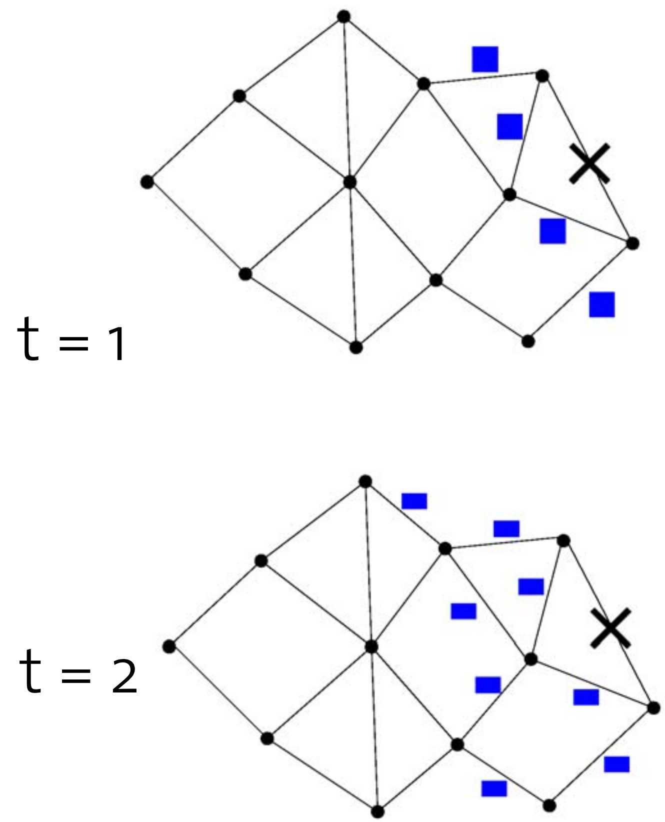
Initial failure



Stationary model

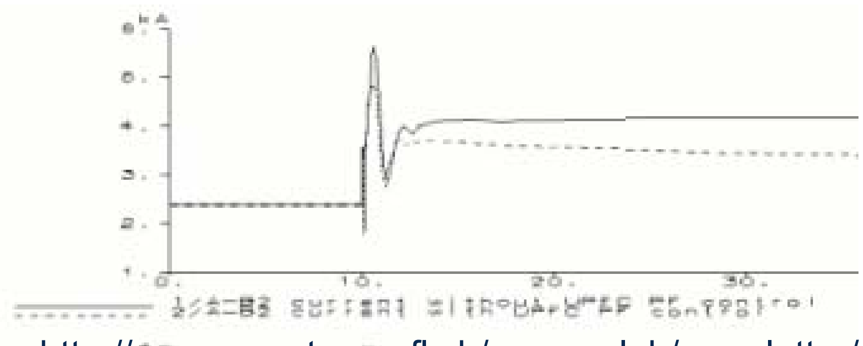


Dynamic model

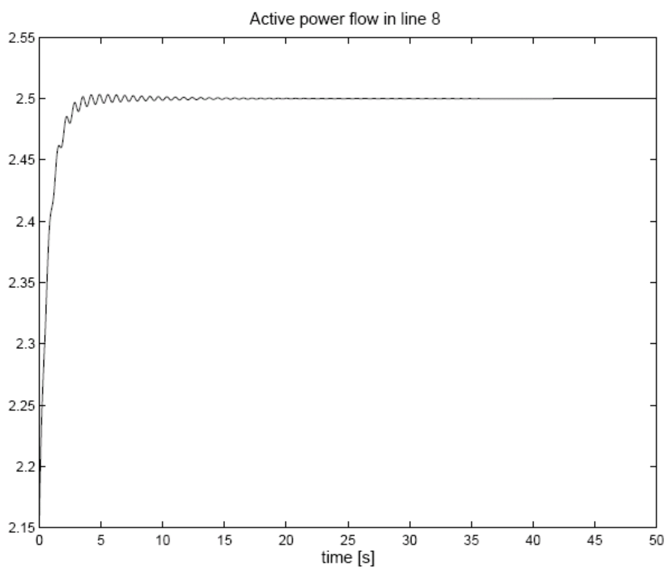


Model Dynamics

Power grid simulation model

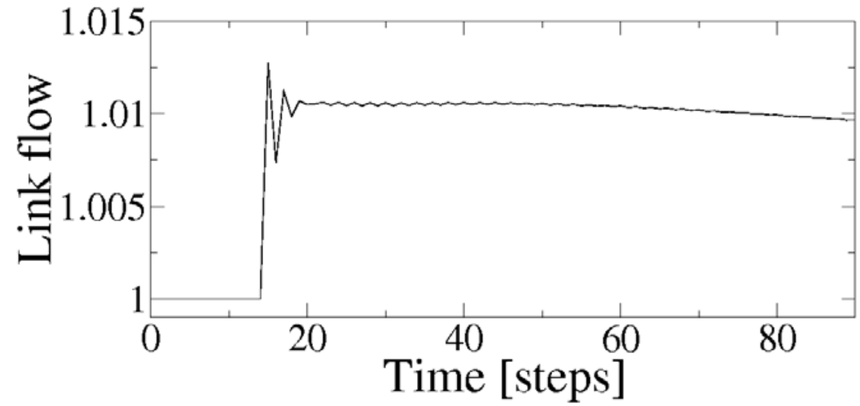


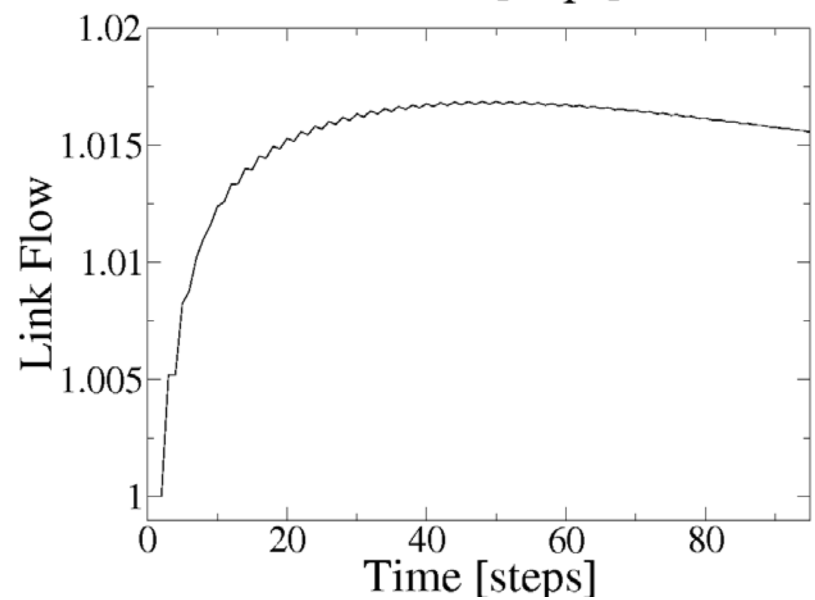
http://wwweurostag.epfl.ch/users club/newsletter/nl8.html



R. Sadikovic: Power flow control with UPFC, (internal report)

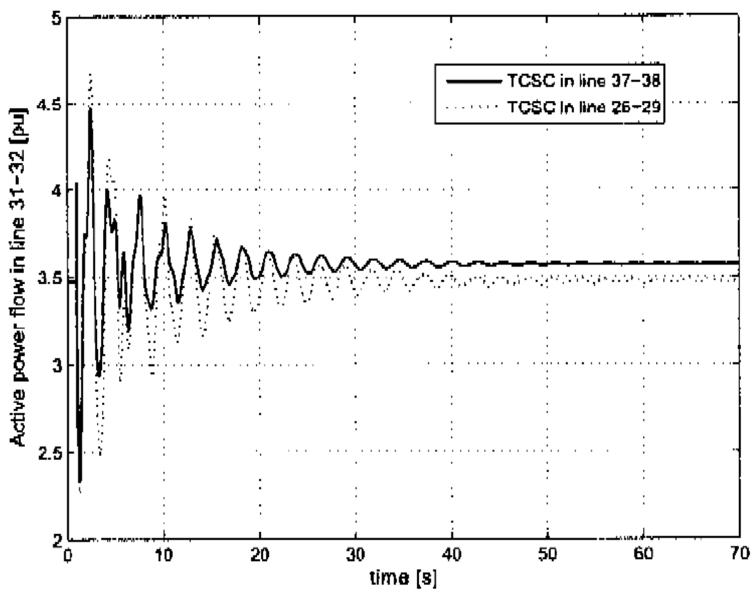
Our model



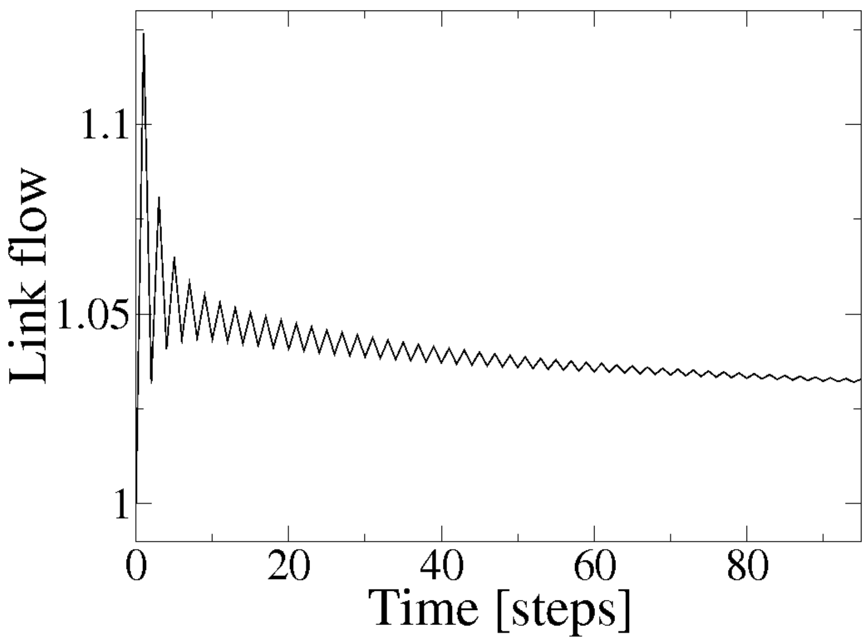


Model Dynamics

Power grid simulation model

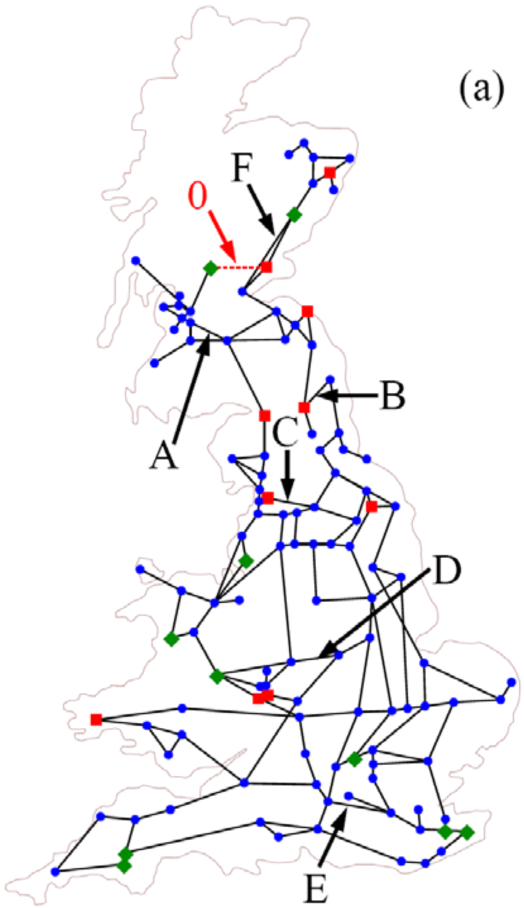


Our model

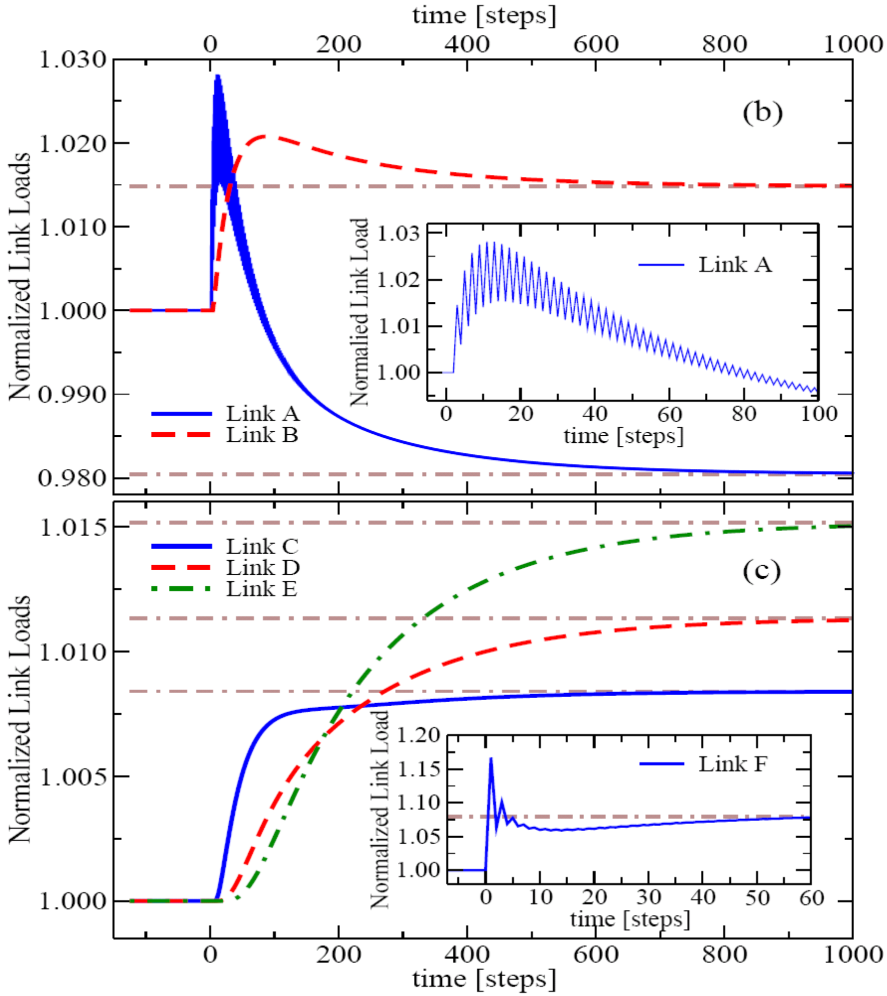


R. Sadikovic: Use of FACTS devices for power flow control and damping of oscillations in power systems, 2006, PhD thesis, ETH Zurich

Model Dynamics



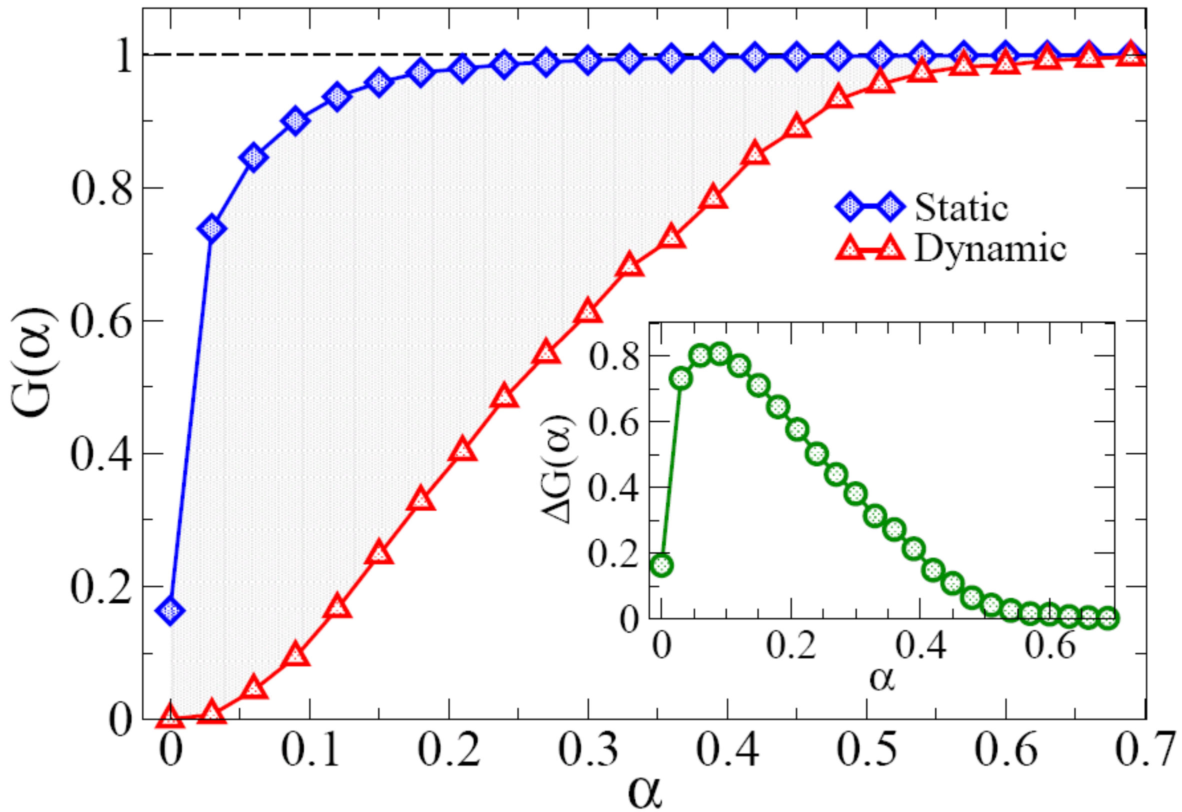
UK high voltage power grid topology (300-400 kV)



Stationary Model vs. Dynamic Model

Link capacities:

$$\mathcal{C}_{ij} = (1+\alpha)\, L_{ij},$$



$|\mathcal{N}|$ number of nodes

$|\mathcal{L}|$ number of links

$|\mathcal{N}_R|$ number of remaining nodes

$|\mathcal{L}_R|$ number of remaining links

$$G_{\mathcal{L}}(\alpha) = \frac{|\mathcal{L}_R|}{|\mathcal{L}|} \approx G_{\mathcal{N}}(\alpha) = \frac{|\mathcal{N}_R|}{|\mathcal{N}|} = G(\alpha)$$

Conclusions

- We have developed models to represent causal interrelationships triggering cascading disaster spreading, allowing to compare the effectiveness of alternative response strategies
- A time-dependent model of disaster spreading allowed us to describe the impact of the topology of interrelationship networks on the spreading dynamics
- The efficiency of different disaster response/relief strategies could be tested by the same model. Different networks require different response strategies! A quick response is crucial.
- Another model has been used to evaluate the vulnerability of freeway networks in different European countries
- A model of cascading failures in power grids showed that stationary spreading models underestimate the robustness of electrical power supply networks by 80% and more.