Safety of Nuclear Power Plants
Tutorial - Dependent Failures (Solution)
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Assume a 2oo4 (2 out of 4) system (system fails when two components fail/system functions when at least three components function) with identical components.

Q1) Calculate the failure likelihood Q_S of the system, while the failure likelihood of the components is given $q_i = 0.01$, $i = 1, 2, 3, 4$.

Q2) Determine system failure likelihood $Q_{S;DF}$. Please take into account dependent failures with the help of the β-factor-model (β = 10%). The observed failures of the components lead to the failure likelihood qj = 0.01, j = 1, 2, 3, 4.

Q1)

$$p^4 + 4p^3q + 6q^2p^2 + 4pq^3 + q^4 = 1$$

$$Q_S = 6q^2p^2 + 4pq^3 + q^4 = 6q^2(1-q)^2 + 4(1-q)q^3 + q^4 = 3q^4 - 8q^3 + 6q^2$$

If all failures are independent of each other, arises with $q_i = 0.01$:

$$Q_S = 5.92 \cdot 10^{-4}$$

Q2) This is a question regarding failure probability considering the DF.

For 'm-out-of-n-system' it is generally: $Q_t = Q_1 + Q_n$

Definition of the β – factor:

$$\beta = \frac{Number\ of\ DF}{Number\ of\ all\ failures} \qquad \beta = \frac{Q_n}{Q_1 + Q_n} = \frac{Q_n}{Q_t}$$

Finally,

$$Q_k = \begin{cases} (1-\beta)\cdot Q_t & k = 1 \\ 0 & m > k > 1 \\ \beta \cdot Q_t & k = n \end{cases}$$

$$Q_{S;DF} = (3Q_1^4 - 8\underbrace{Q_1^3}_{\boxed{K=1}} + 6Q_1^2) + \underbrace{Q_4}_{\boxed{K=4}} \approx 6Q_1^2 + Q_4$$

$$Q_k = \begin{cases} (1-\beta)Q_t, & \text{if } k = 1 \\ \beta Q_t, & \text{if } k = n = 4 \end{cases}$$

$$Q_{S;DF} \approx 6Q_1^2 + Q_4 = 6(1-\beta)^2 Q_t^2 + \beta Q_t$$

$$Where\ Q_t = 0.01\ and\ \beta = 0.1$$

(Qt is the total failure likelihood of one component)

Therefore,

$$4.86 \times 10^{-4} + 10^{-3} \approx 1.483 \times 10^{-3}$$